A Model for Local Plasticity Effects on Fatigue Crack Growth

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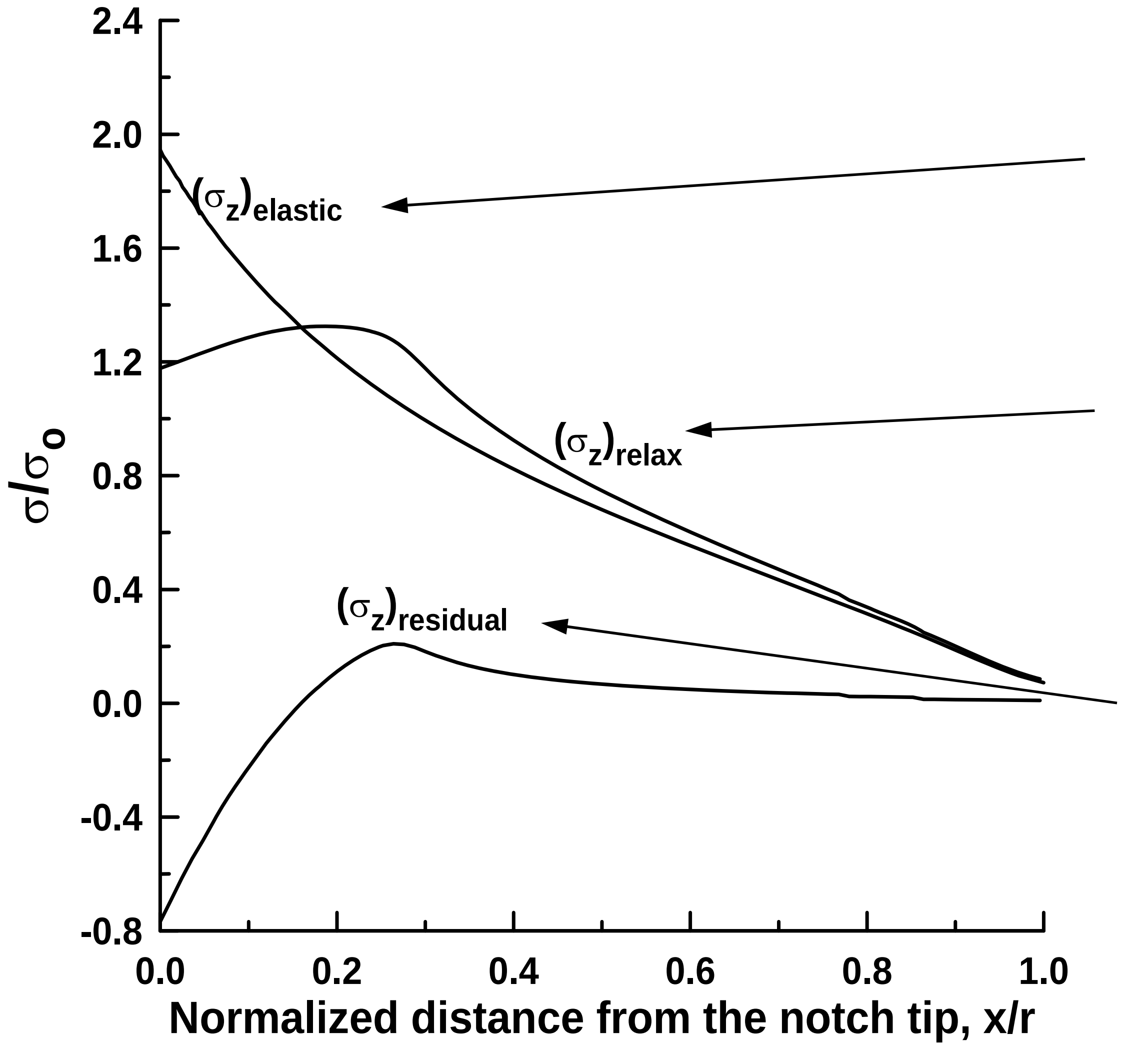
Potential Plasticity Effects at Stress Concentrations

- When applied loading causes local yielding at stress concentrations, the resulting plasticity can have several impacts on FCG rates
  - Local yielding causes residual stresses
  - Residual stresses cause local changes in R ratio
  - Plasticity can also influence load interaction effects
  - LEFM parameters (ΔK) may not accurately describe crack driving force if yielding is severe (cyclic plasticity)

Outline

- **Model Development**
  - Cyclic shakedown model
  - Weight function stress intensity factor solutions
  - Crack closure model
  - *J*-integral estimates for cracks at holes
- **Experimental Evaluation**

Shakedown Analysis and Residual Stress



Perform linear elastic stress analysis

Perform elastic-plastic stress analysis based on linear elastic results to obtain relaxed stress

Determine the shakedown residual stress

$$\sigma_z^{residual}(x) = \sigma_z^{relaxed}(x) - \sigma_z^{elastic}(x)$$

Elastic-Plastic Relaxation: Stress Relaxation & Load Redistribution

Calculate plastically relaxed stress at location *x* by applying Neuber's rule

$$\sigma_{equiv}^{elas} \times \varepsilon_{equiv}^{elas} = \sigma_{equiv}^{relax} \times \varepsilon_{equiv}^{relax} \qquad \left(\sigma_{equiv}^{relax}\right)^2 + \alpha \frac{\left(\sigma_{equiv}^{relax}\right)^{m+1}}{\sigma_o^{m-1}} - \left(\sigma_{equiv}^{elas}\right)^2 = 0$$

Determine normal stress components from relaxed equivalent stress

$$\left(\sigma_j^{relax}\right) = \frac{\left(\sigma_j^{elas}\right)}{\left(\sigma_{equiv}^{elas}\right)}\left(\sigma_{equiv}^{relax}\right), \quad j = x, y, z$$

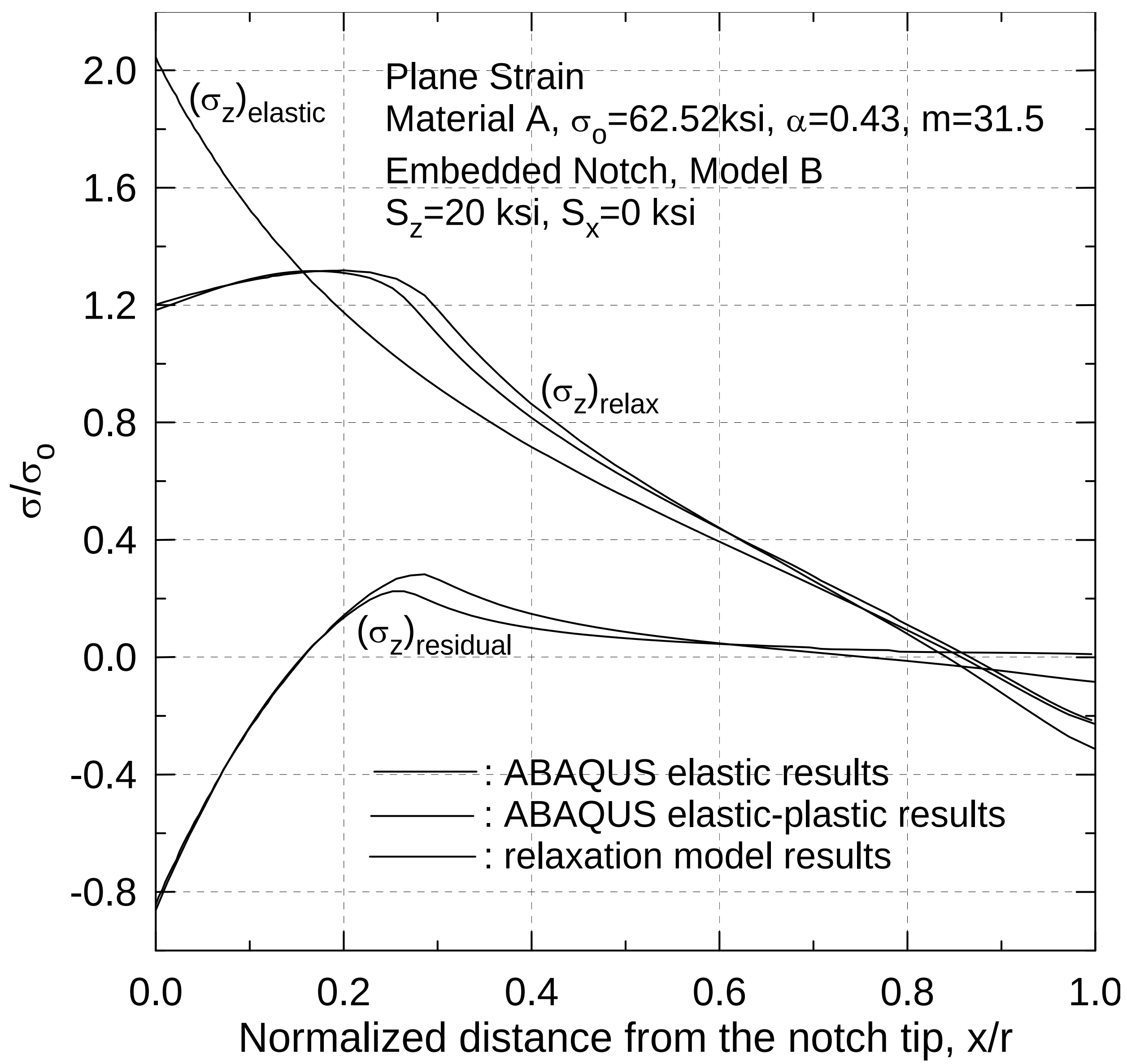
Redistribute increment of local load at position *x* resulting from local stress relaxation

$$\Delta L(x) = \Delta x[\sigma_z^{elastic}(x) - \sigma_z^{relaxed}(x)]$$

Distribute increment of global load needed to maintain force balance over load bearing area

$$\Delta L_{global} = \int_{\substack{load\\bearing\\area}} [\sigma_z^{elastic} - \sigma_z^{relaxed}]d(area)$$

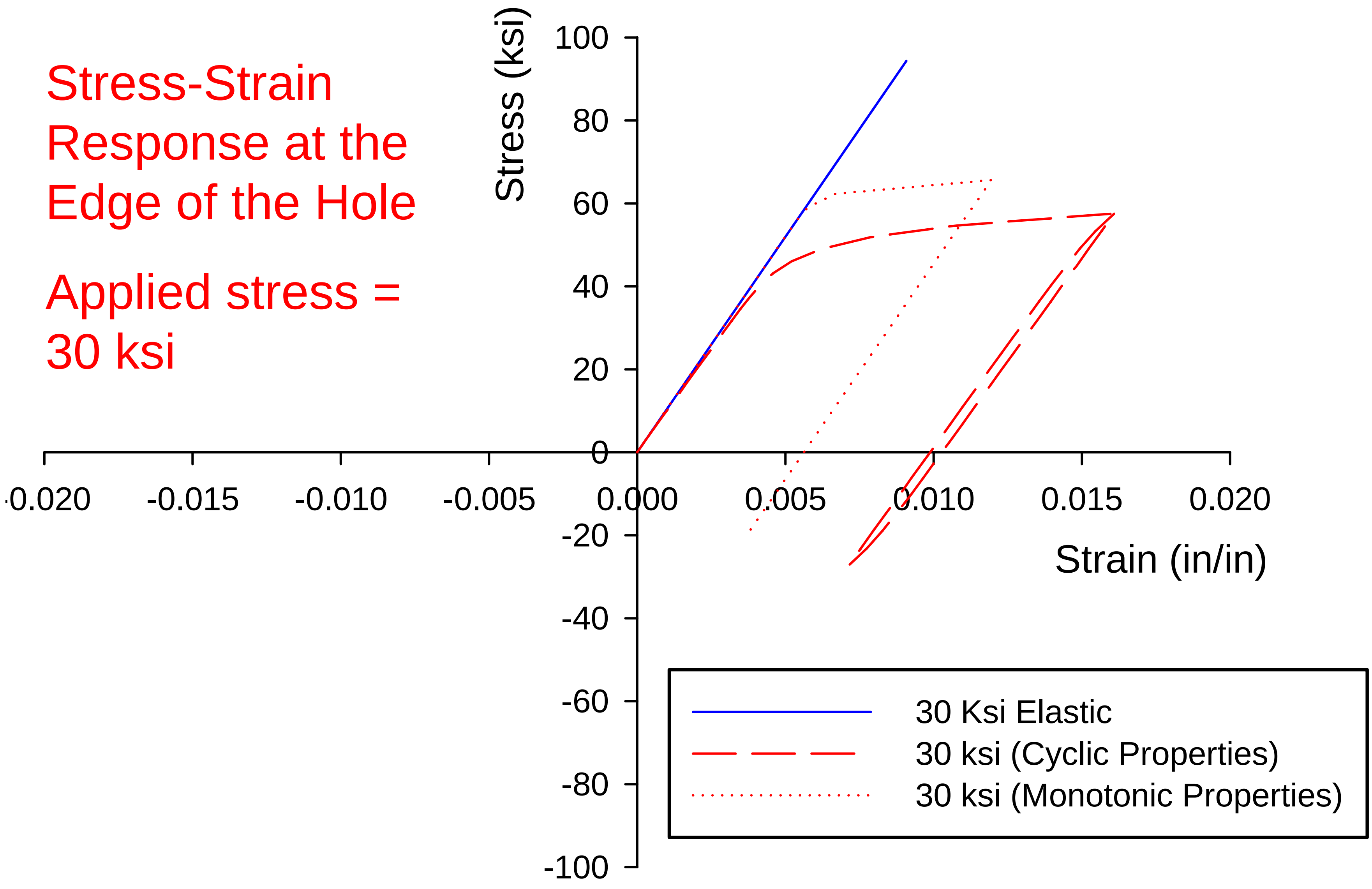
Finite Element Verification of Shakedown Analysis



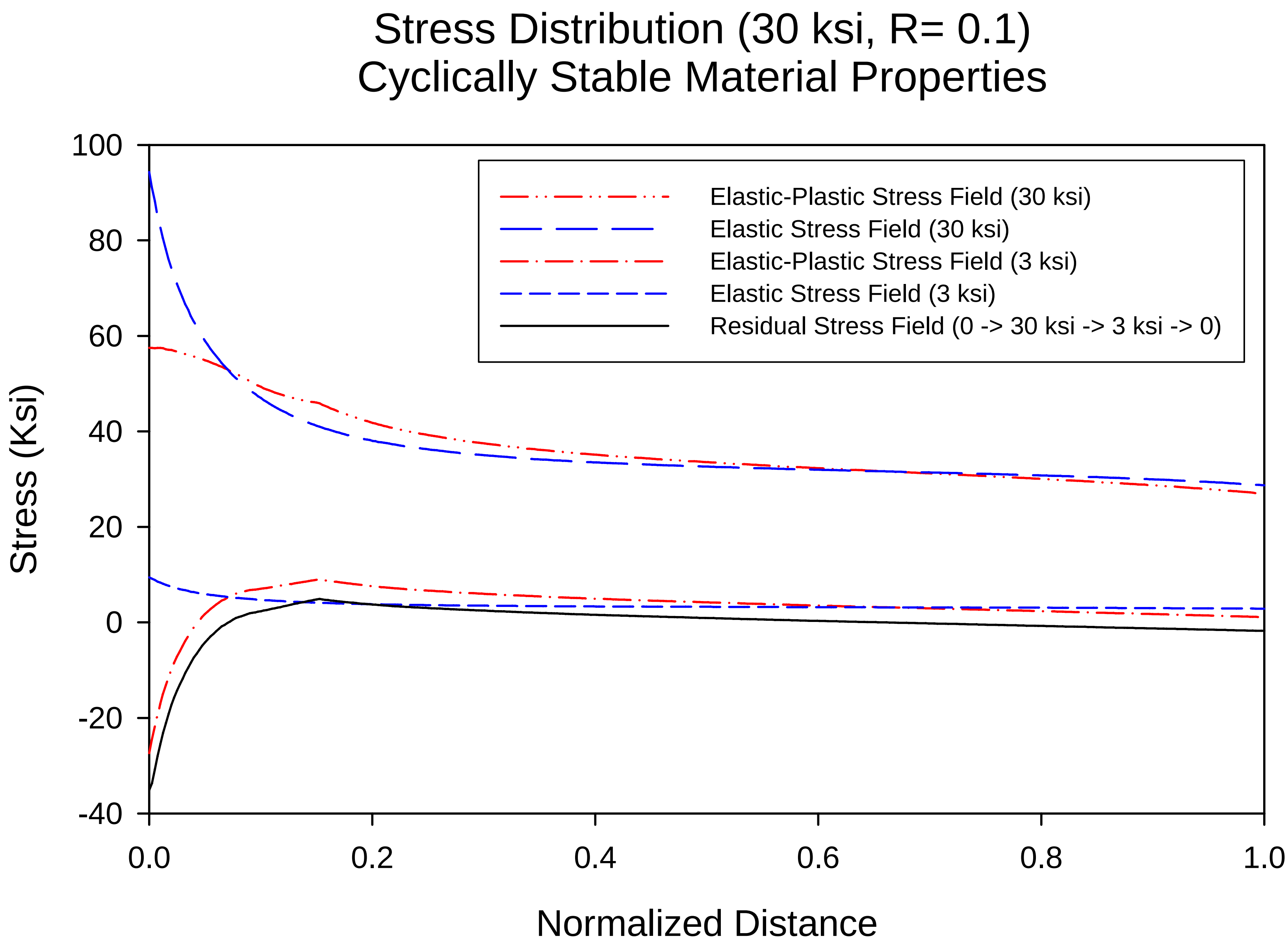
Shakedown Behavior (*R* = 0.1)

Stress-Strain Response at the Edge of the Hole

Applied stress = 30 ksi



Shakedown Behavior (*R* = 0.1)



Weight Function *K* Solutions for Cracks at Holes

- Historical *K* solutions for cracks at holes only accept remote tension/bend or pin loading
- Weight function solution is needed to address arbitrary stress distributions
- Approach
  - Identify a suitable univariant weight function formulation
  - Use FADD-3D boundary element code to generate reference solutions

Univariant Weight Function Method for Cracks at Holes

- Determine K at a-tip and c-tip by direct integration:

$$K_{a,c} = \int_0^c W_{a,c}\,\sigma(x)\,dx$$

- Weight function at the a-tip is

$$W_a = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1a}\sqrt{\frac{x}{c}} + M_{2a}\cdot\frac{x}{c} + M_{3a}\left(\frac{x}{c}\right)^{\frac{3}{2}}\right]$$

- M-factors are given by

$$M_{1a} = \frac{\pi}{\sqrt{4Q}}\left(30F_1 - 18F_0\right) - 8$$

$$M_{2a} = \frac{\pi}{\sqrt{4Q}}\left(60F_0 - 90F_1\right) + 15$$

$$M_{3a} = -\left(1 + M_{1a} + M_{2a}\right)$$

- Q is the shape factor,

$$Q = \begin{cases} 1 + 1.464\left(a/c\right)^{1.65}, & a/c \le 1 \\ 1 + 1.464\left(a/c\right)^{-1.65}, & a/c > 1 \end{cases}$$

Glinka (1991, 1998)

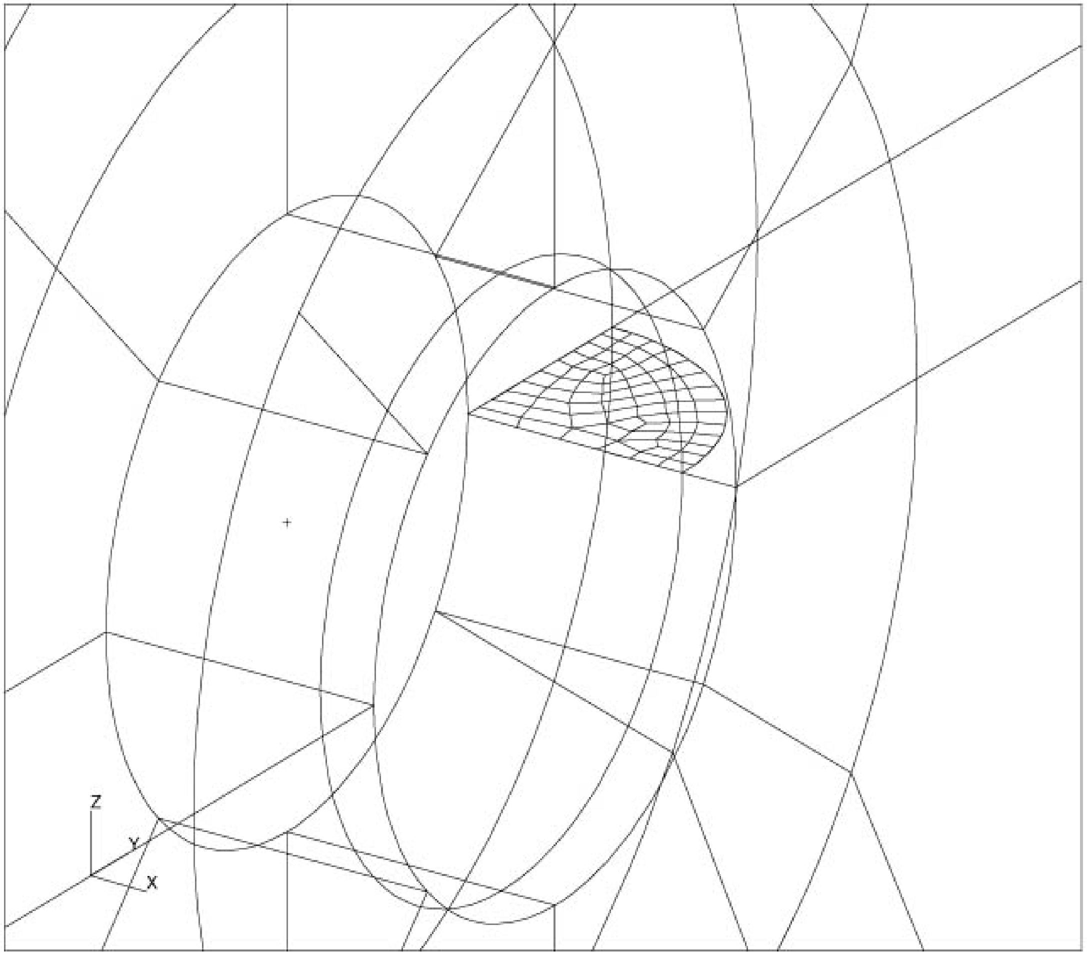
σ(*x*) is the stress distribution on the crack surface in the <u>uncracked</u> body

F_0, F_1 are normalized reference solutions

0: uniform tension
1: linear bend stress

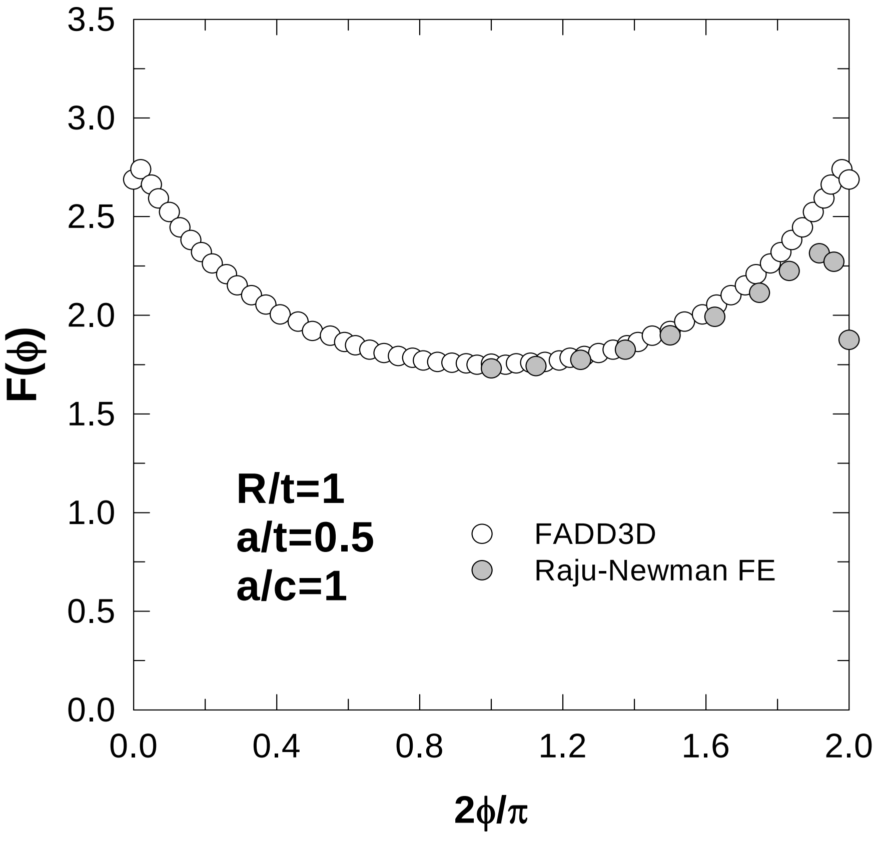
Reference Solutions from FADD-3D Analysis

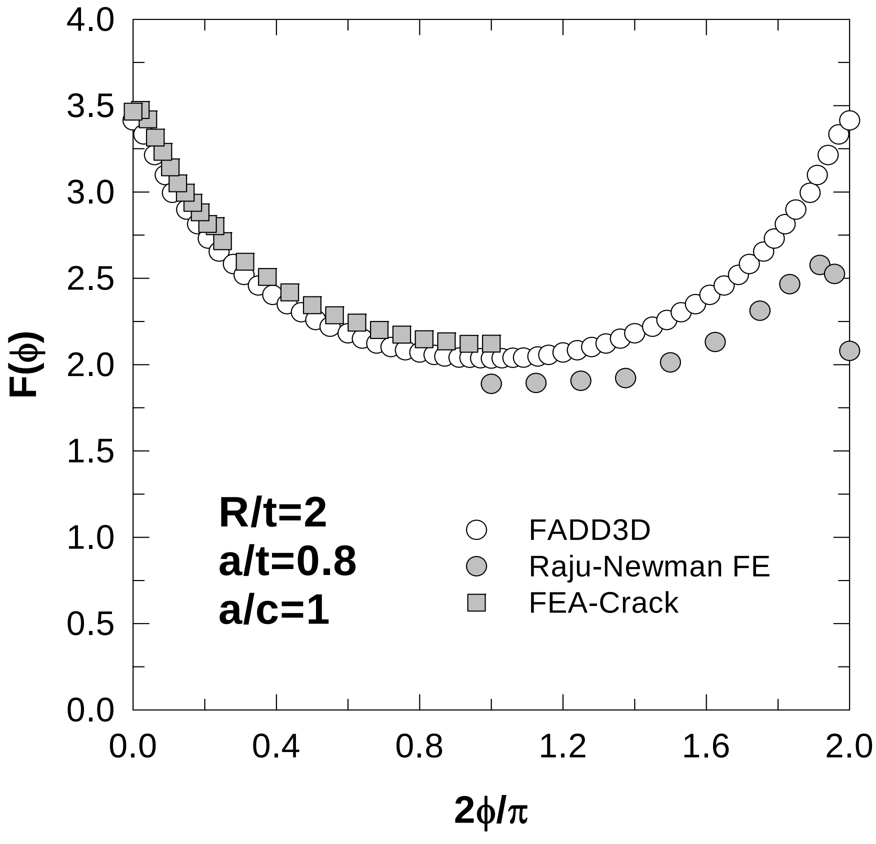
- Total of ~150 geometry models required for surface and corner cracks
  - *a/c* = 0.5, 1.0, 2.5, 5, 10
  - *a/t* = 0.1, 0.2, 0.5, 0.8, 0.9
  - *R/t* = 0.25, 1, 2
- Two reference solutions (uniform tension, linear stress gradient) per geometry
  - Independent check against univariant stress gradient to validate
  - All solutions agree within few percent



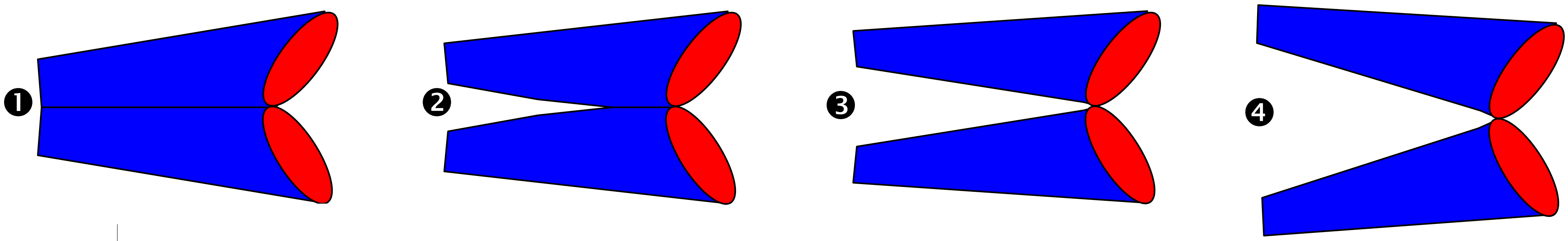
Evaluation of FADD-3D Solutions

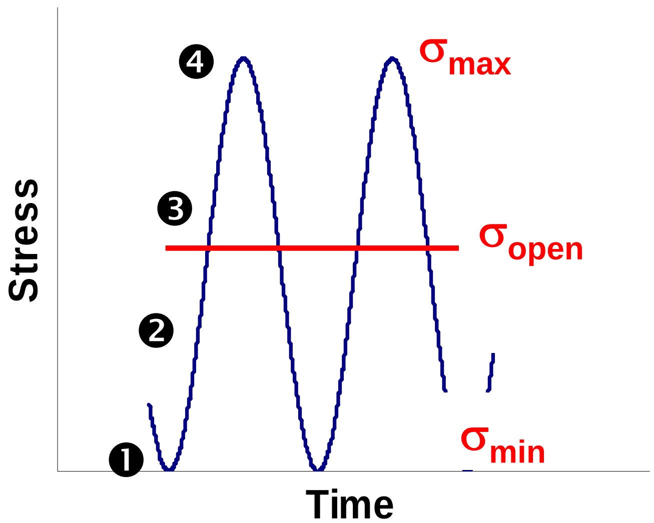
Center surface crack at a hole under uniform remote tension





Crack Closure

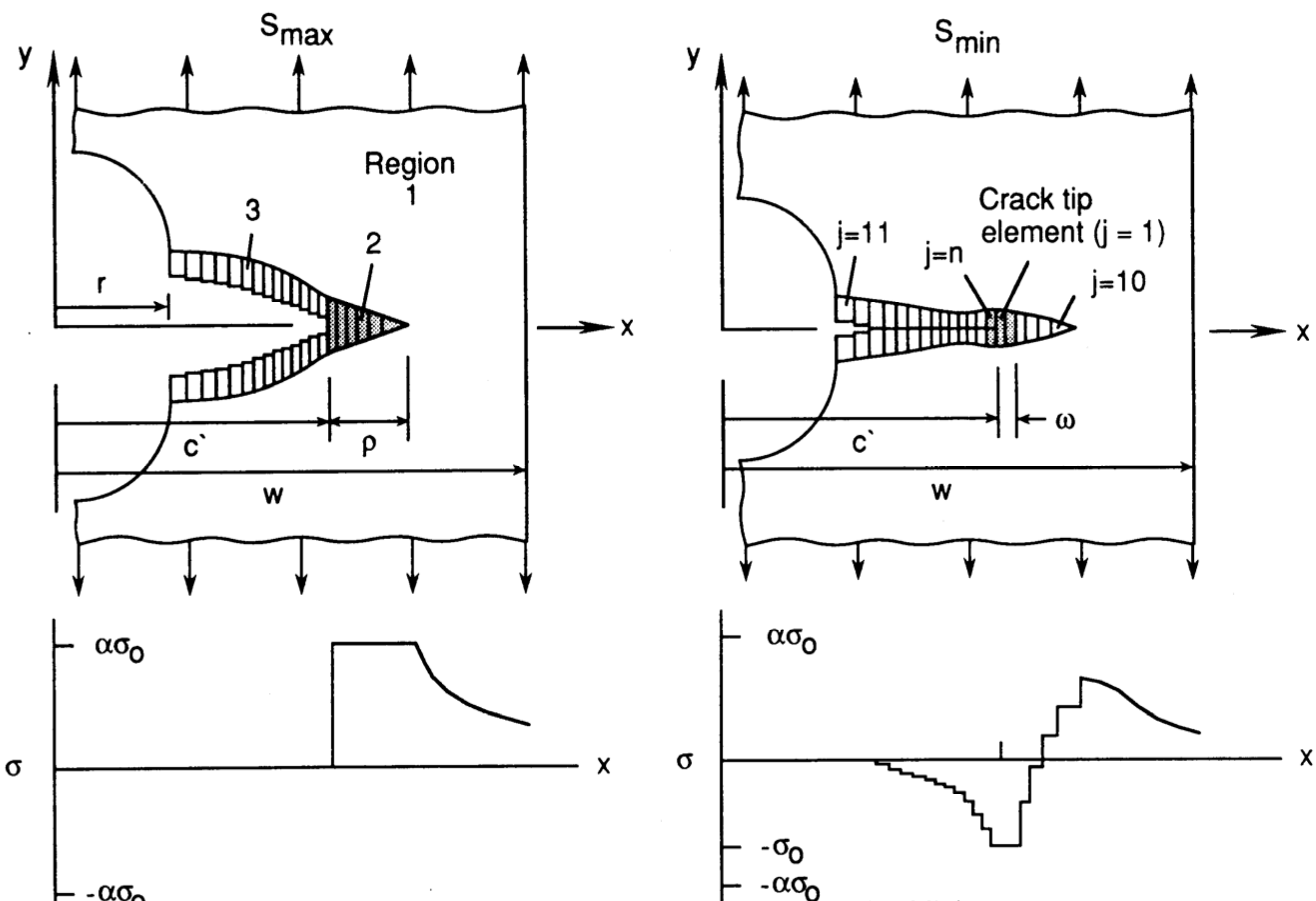




$$\Delta K_{eff} = \left(K_{\max} - K_{open}\right) = \left(\frac{\sigma_{\max} - \sigma_{open}}{\sigma_{\max} - \sigma_{\min}}\right)\Delta K$$

- FCG causes residual plastic deformation in crack wake
- Residual deformation affected by details of load history
- Residual plastic deformation affects crack driving force for future cycles

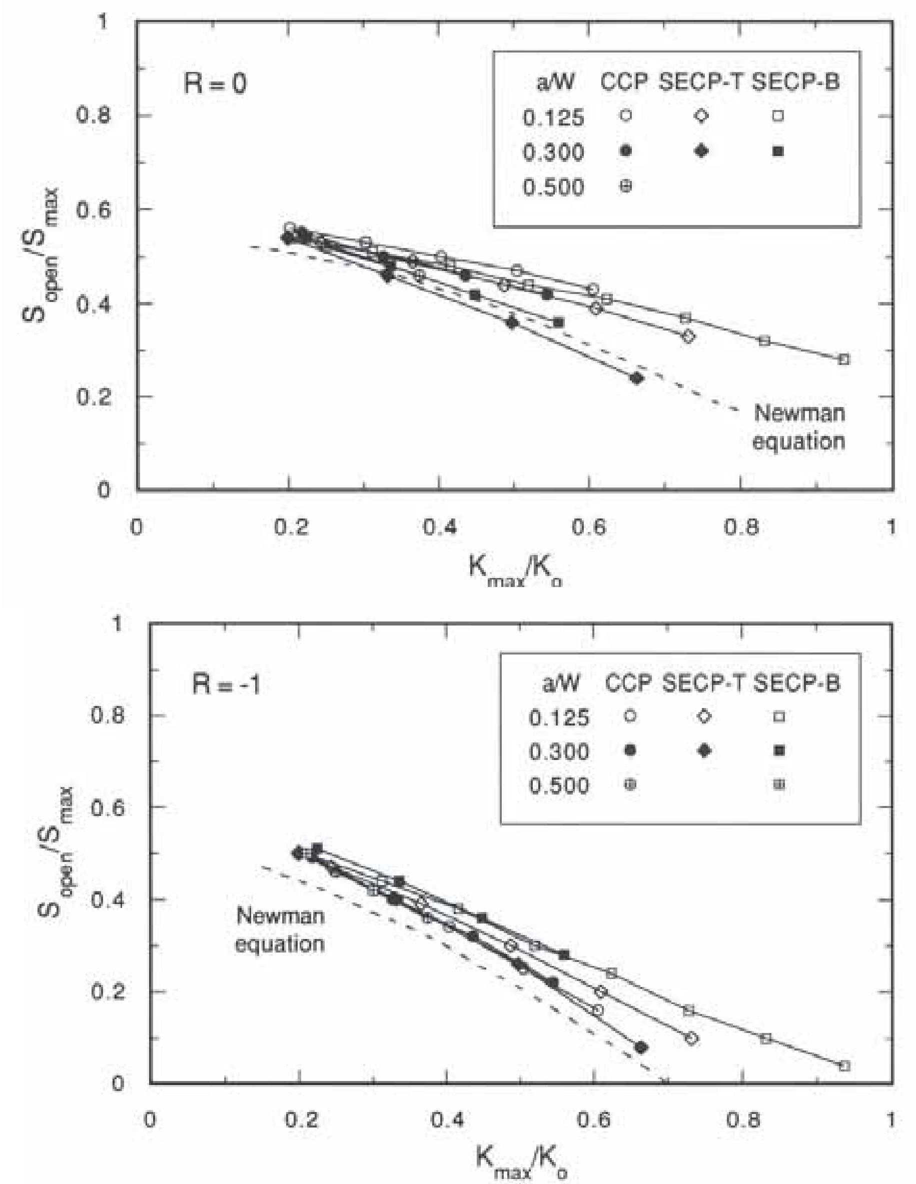
Strip Yield Model for Crack Closure



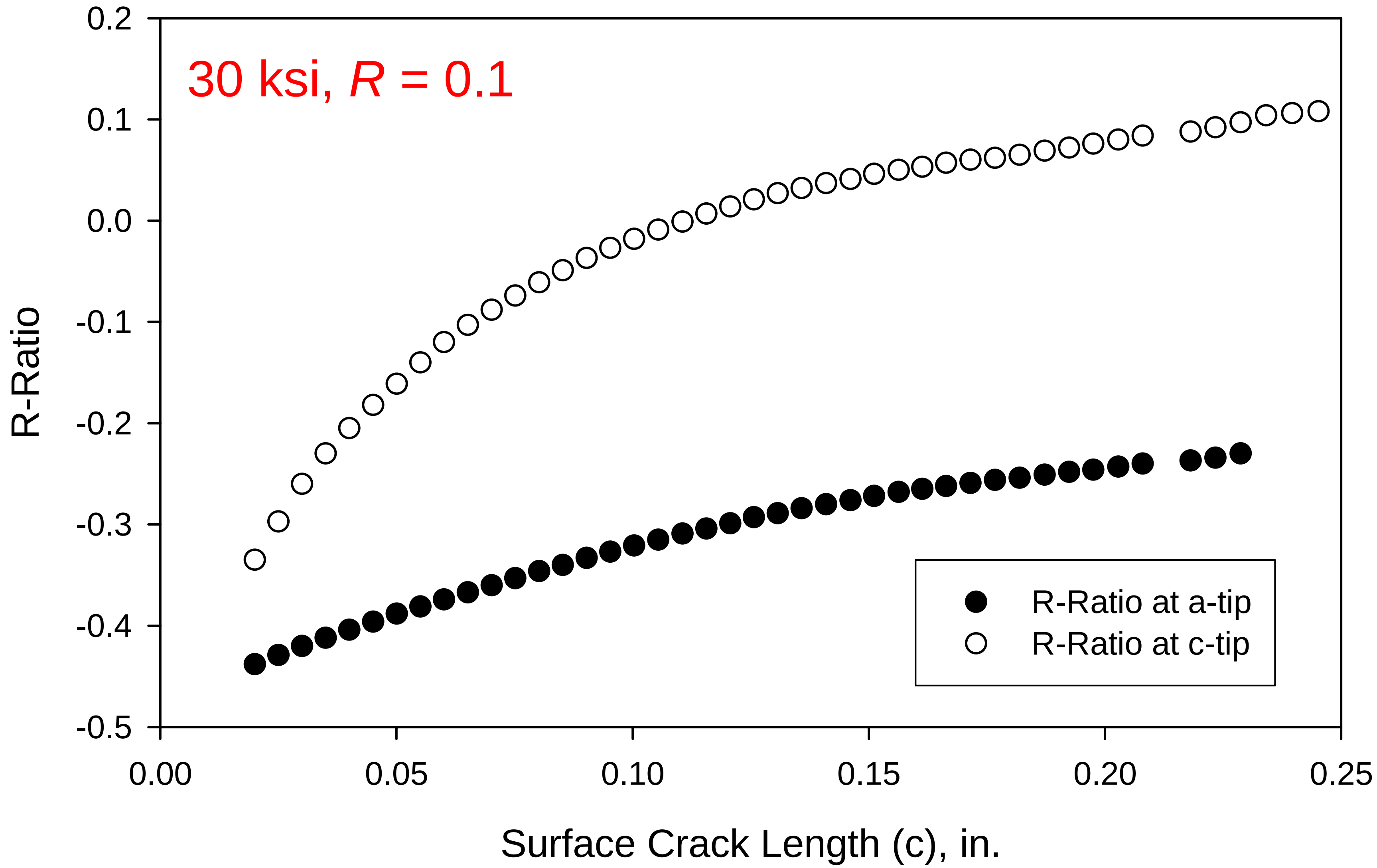
(a) Maximum stress. (b) Minimum stress.

How to Apply Loads to the Strip Yield Model?

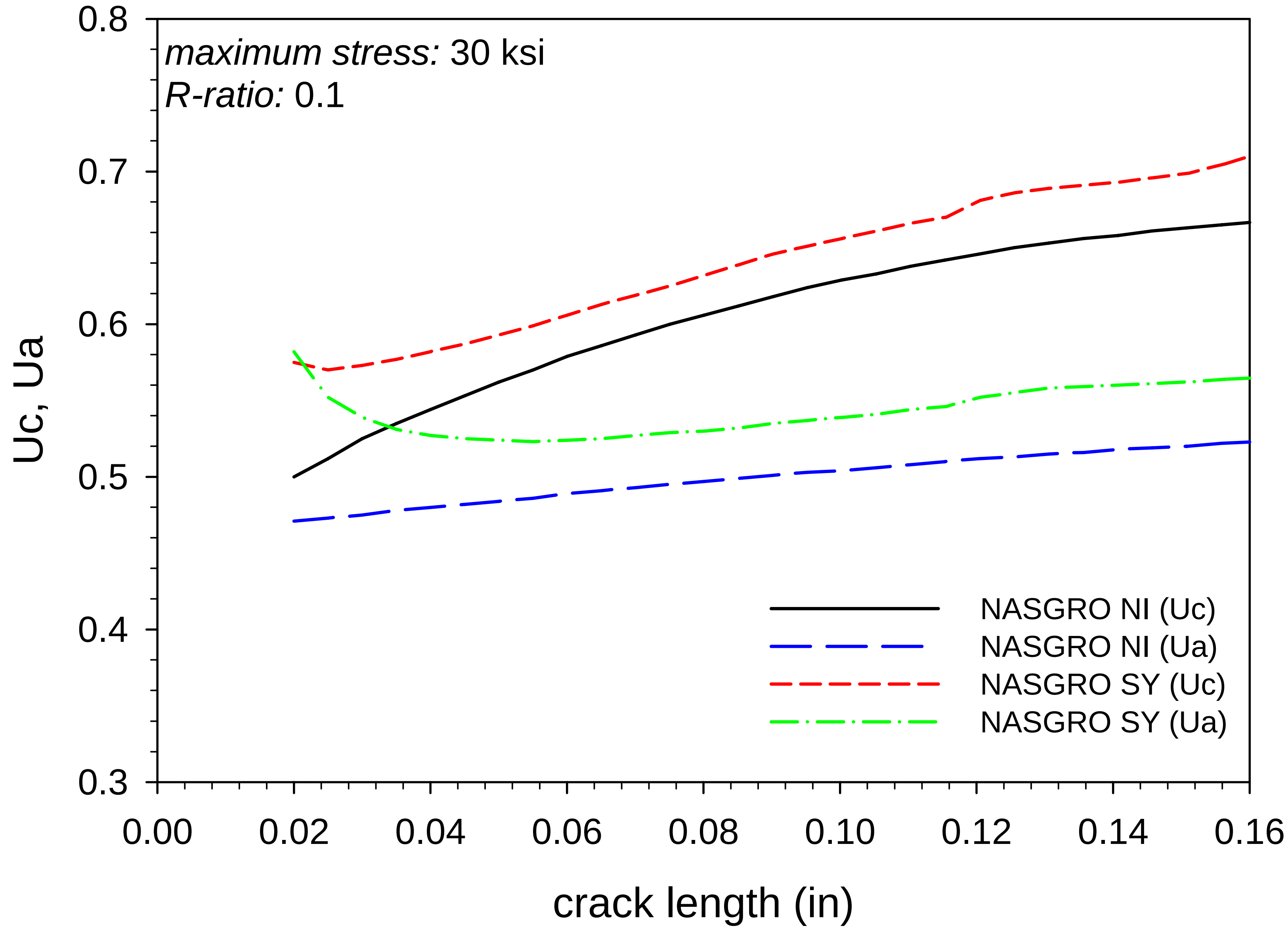
- How to relate arbitrary geometries and loading conditions to the Strip Yield model (center-cracked plate)?
- K-Analogy approach:
  - Same crack length
  - Equivalent tensile stress to give same *K*
- K-Analogy approach is supported by detailed finite element studies of crack closure (McClung, 1994)
- K-Analogy approach can also address effects of superimposed residual stresses



Local Changes in *R*-Ratio Due to Shakedown



Crack Opening Stresses at the *a*-tip and *c*-tip



J-Integral for Cyclic Plasticity

- ΔK is no longer an accurate description of the crack driving force when <u>cyclic</u> plasticity occurs in the uncracked body near the crack location
  - Monotonic plasticity followed by elastic cycling appears to be adequately addressed by *K* with shakedown methods
- Best available engineering description of the elastic-plastic crack driving force is the range of the *J*-integral, ΔJ

Engineering Estimate for *J*

- Total *J* = Elastic *J* + Plastic *J*

$$J_a(a, c, P, \theta) = J_{a_e}(a_e, c, P, \theta) + J_{a_p}^{RSM}(a, c, P, \theta)$$

$$J_c(a, c, P, \theta) = J_{c_e}(a, c_e, P, \theta) + J_{c_p}^{RSM}(a, c, P, \theta)$$

- Fully plastic *J* usually negligible for cracks at holes
- First order plastic estimate of Elastic *J* calculated from Elastic *K* with effective crack size

$$J_{a_e}\left(a_e, c, P, \theta\right) = \left[\frac{\left(K(a_e, c, P, \theta)\right)^2}{E'}\right]$$

$$J_{c_e}\left(a, c_e, P, \theta\right) = \left[\frac{\left(K(a, c_e, P, \theta)\right)^2}{E'}\right]$$

  - Effective crack length

$$a_e = a + \phi\, r_y(a)$$

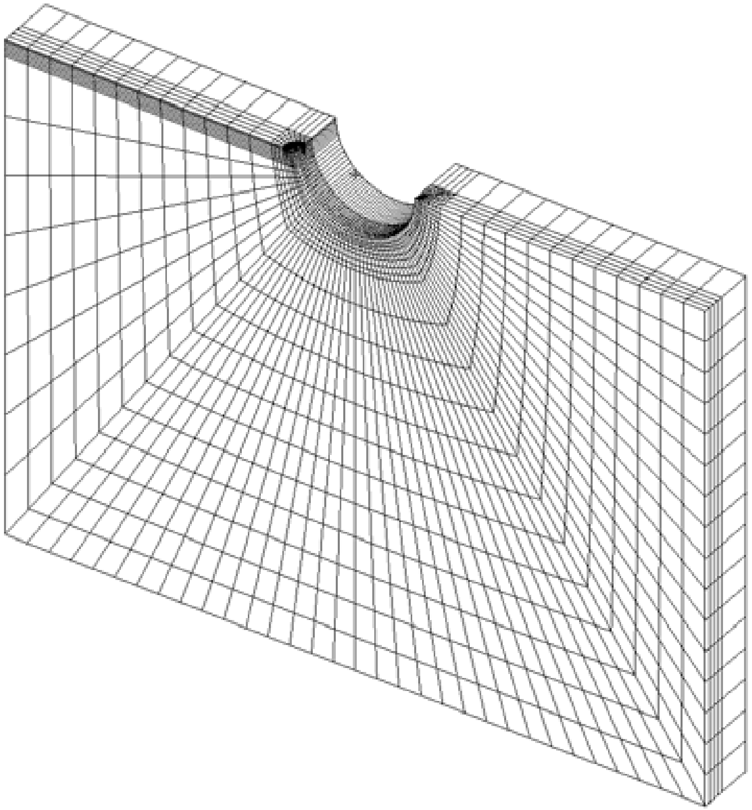
$$c_e = c + \phi\, r_y(c)$$

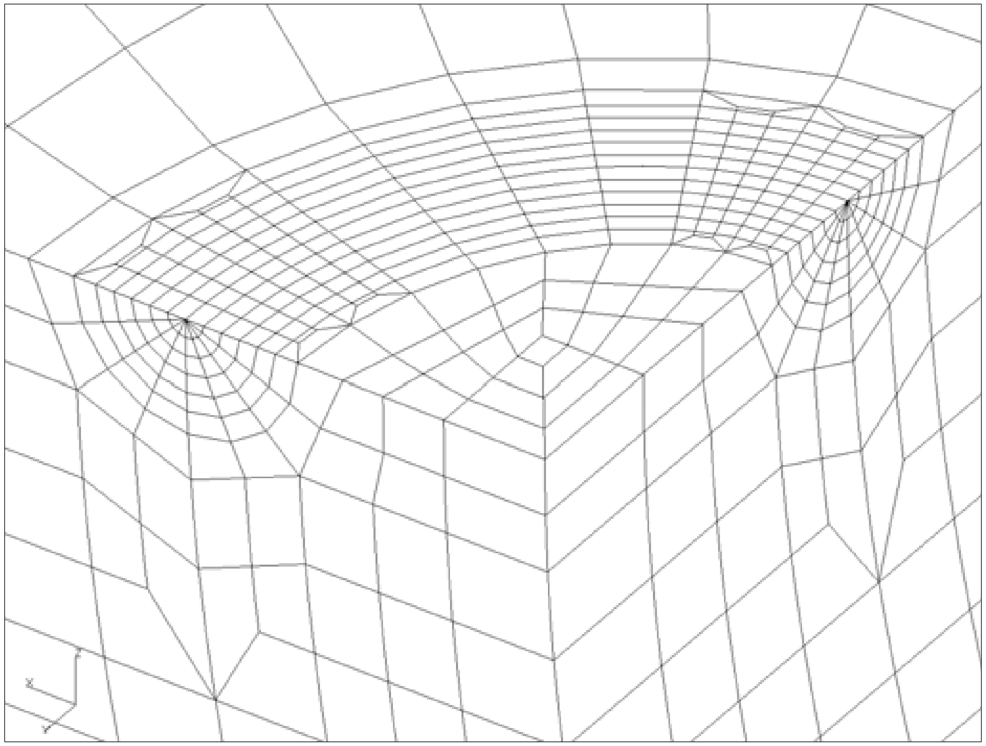
Practical Notes about *J*

- ΔJ for cracks at holes is estimated from the
  - Linear elastic value of ΔK
  - Based on the elastic stress field (not the shakedown field)
  - Small first-order plastic correction on effective crack size
- ΔJ is significantly different from ΔK only for cyclic plasticity
- ΔJ can be implemented in an existing LEFM engineering method by calculating and applying an equivalent ΔK value
  - $\Delta K_{equiv} = (E' \Delta J)^{1/2}$, where $E' = E/(1-\nu^2)$ for plane strain

Evaluation of *J* Estimation Methods

- Generate total *J* solutions for corner cracks at holes using elastic-plastic finite element methods (FEA-Crack)
- Compare *J*-estimation methods against FEA-Crack results





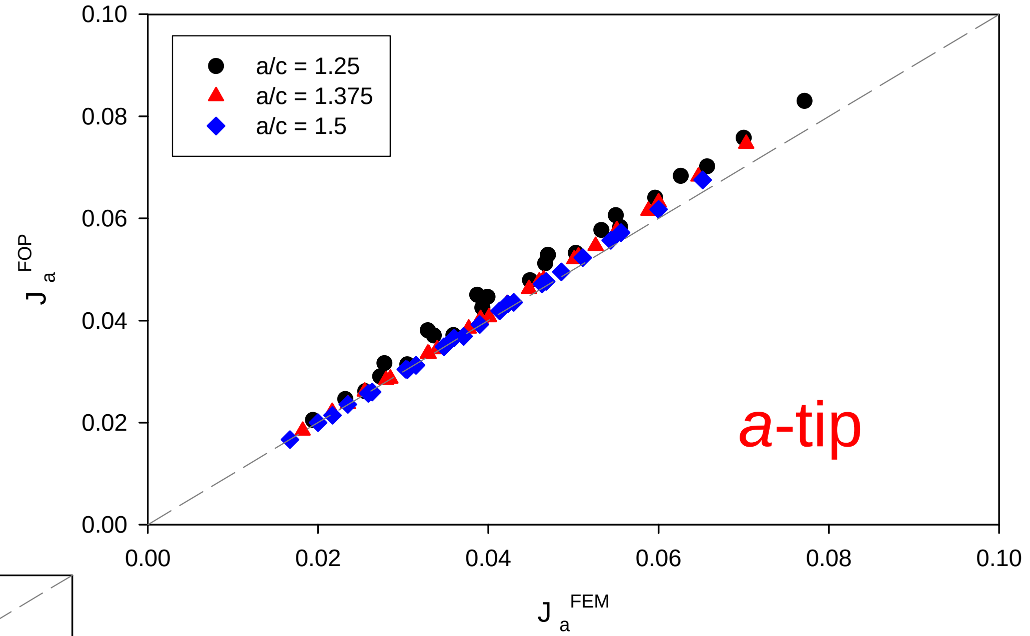
J-Integral Estimates for Corner Crack at Hole (*a*-tip, *c*-tip)

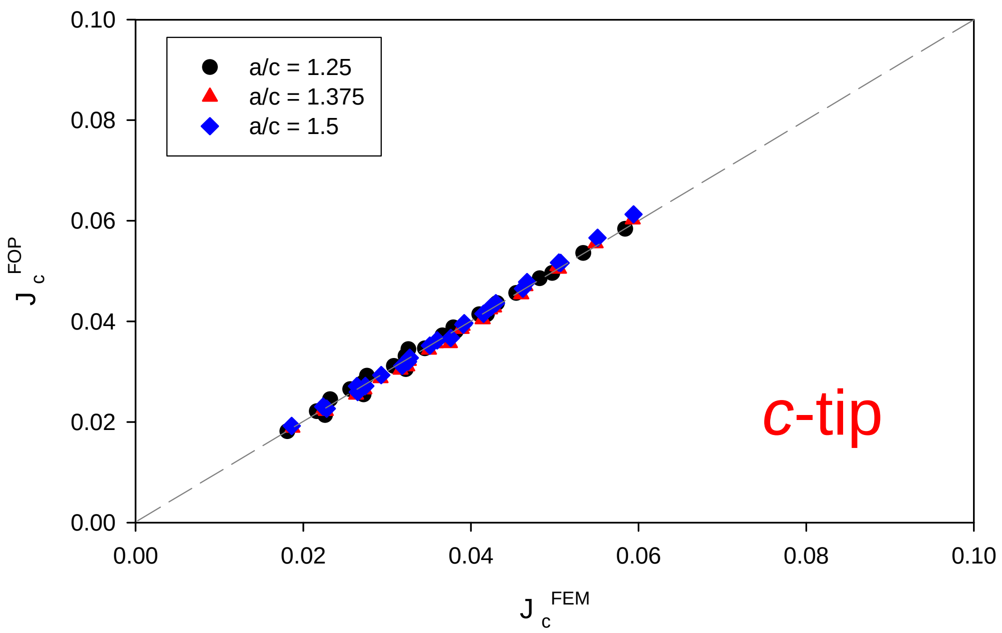
Comparisons of

first-order plastic estimate

vs.

elastic-plastic FE analysis





3 crack aspect ratios

8 crack sizes

3 remote loads

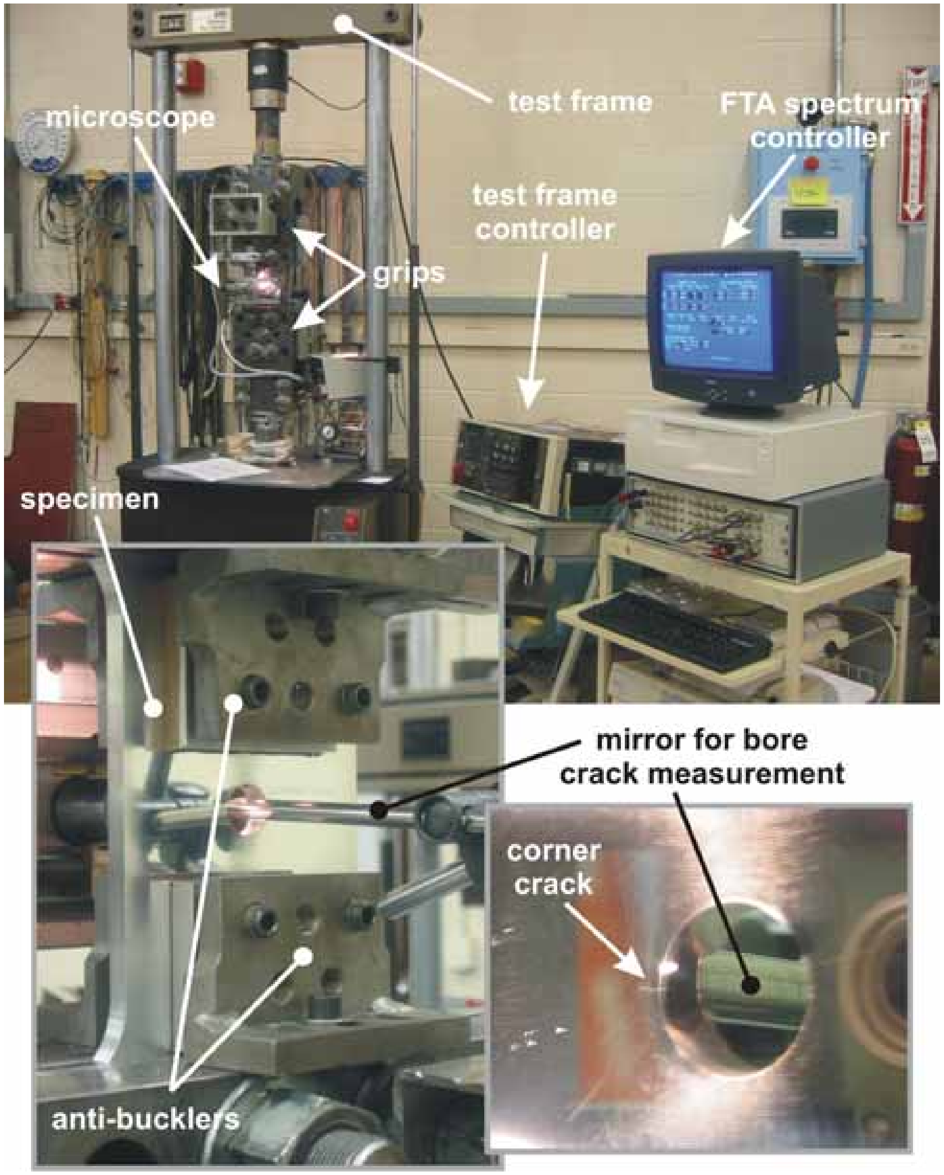
2 material models

Implementation of Analytical Model

- Elastic-Plastic FCG Model implemented in a custom version of NASGRO®
  - Some features are in the current production version of NASGRO
    - Cyclic shakedown model
    - Weight function SIF solutions
  - Some features are not in the current production version of NASGRO
    - The *K*-analogy approach was implemented as a modification to the current NASGRO strip yield model
    - The *J*-integral estimates were implemented as a modification to the current NASGRO SIF solutions

Experimental Evaluation: Corner-Crack-at-Hole Tests

- Aluminum 2124-T851
- Hole Radius = 0.3” (7.6 mm)
- Thickness = 0.3” (7.6 mm)
- Width = 3.5” (89 mm)
- Initial crack sizes
  - *a ~ 0.025” (0.635 mm)*
  - *c ~ 0.020” (0.508 mm)*
- Crack lengths measured often during test with optical microscope at both *a*-tip (bore) and *c*-tip (surface)

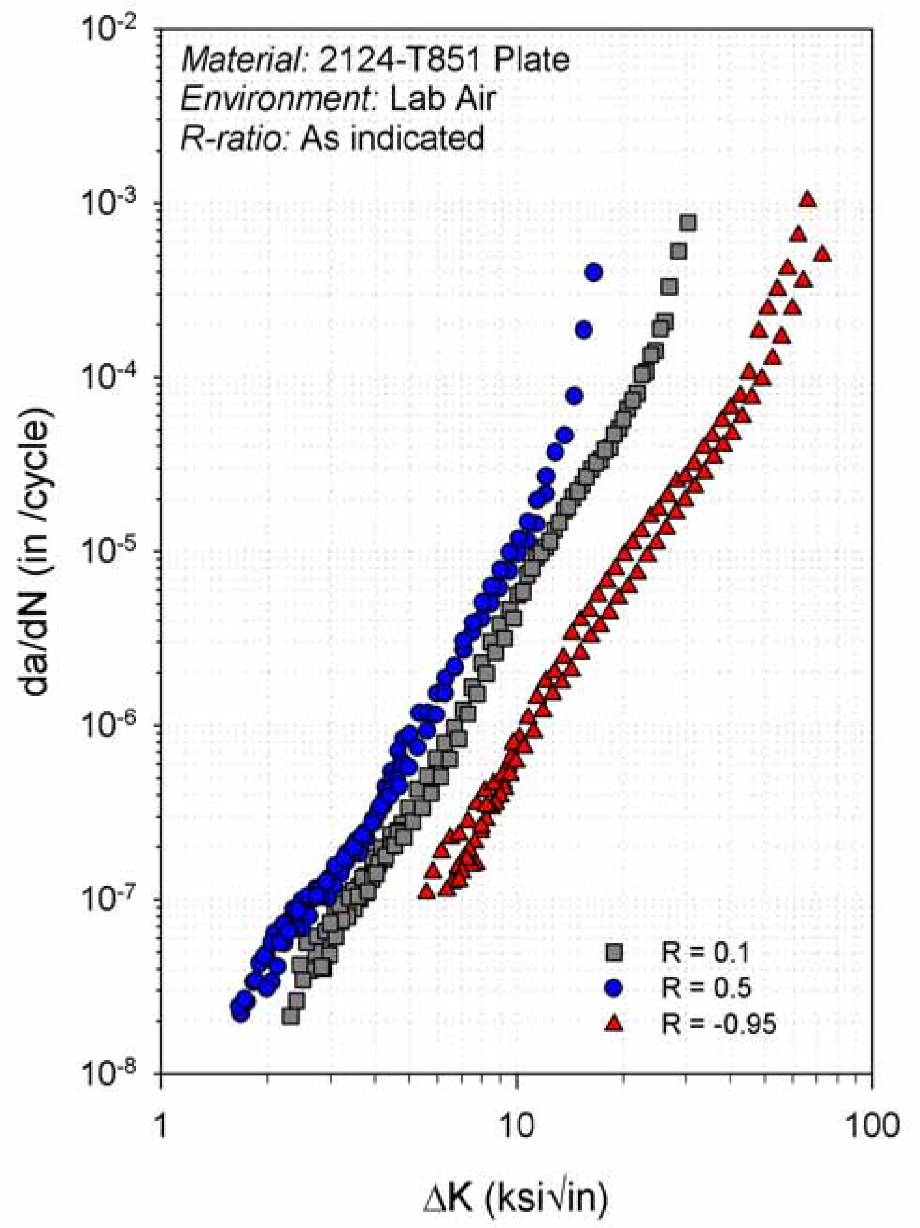


Experimental Evaluation: Test Material

2124-T851 Plate

0.2% YS = 63.7 ksi

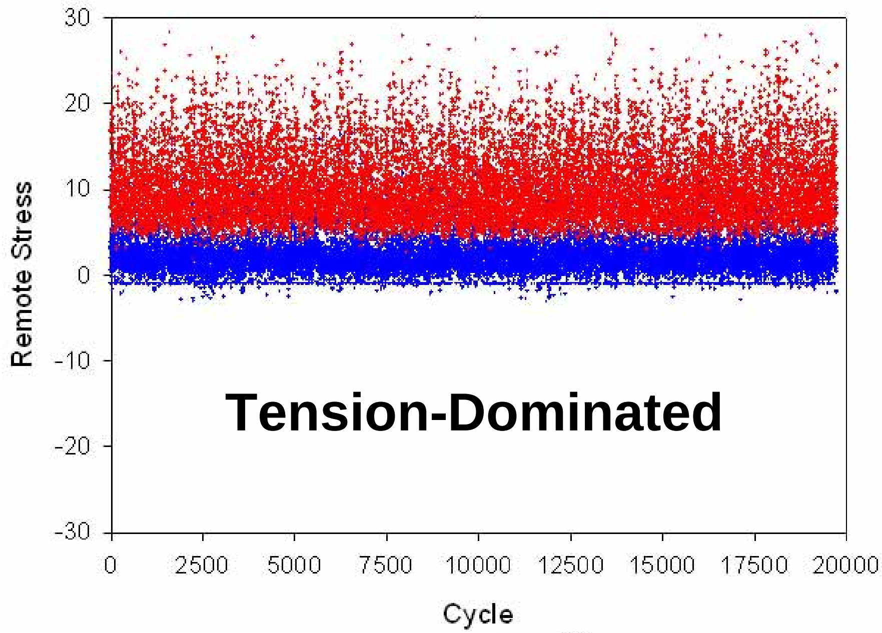
UTS = 69.6 ksi

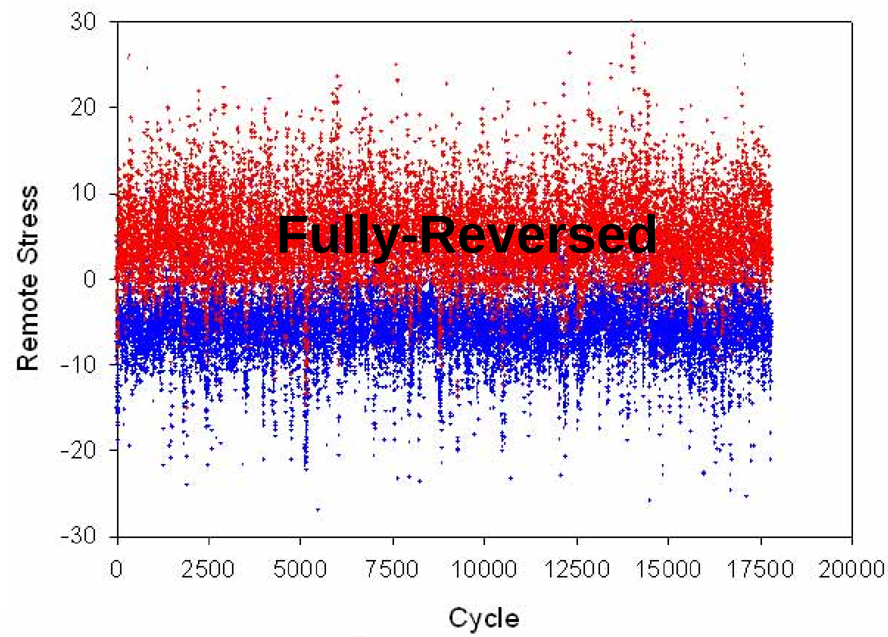


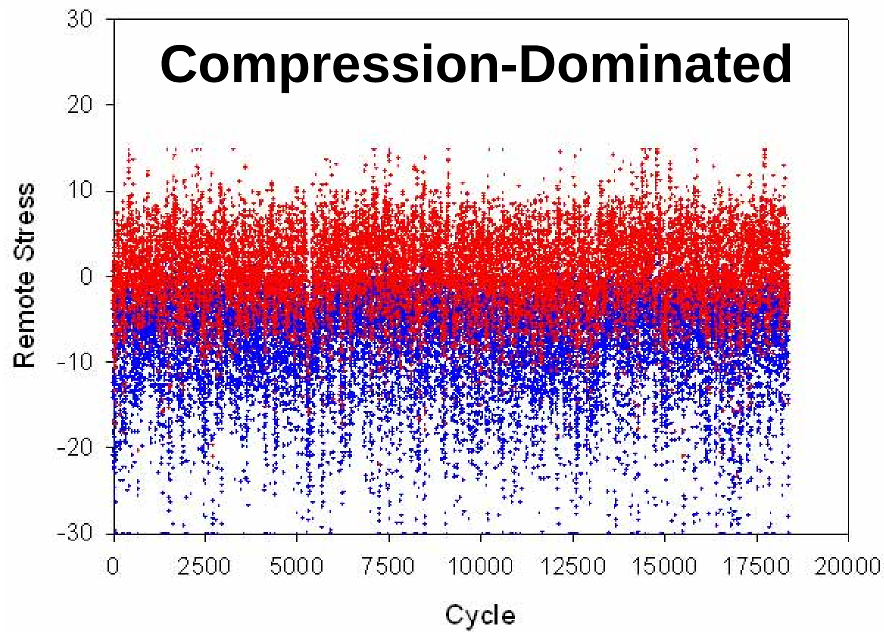
Experimental Evaluation: Load Spectra

- Perform FCG tests with three contrasting load spectra
  - Tension-Dominated Spectrum
  - Fully-Reversed Spectrum
  - Compression-Dominated Spectrum
- Scale the spectra to different magnitudes
  - Maximum applied stress = 30 ksi
    - Local elastic stress = 94.5 ksi
    - Significant plasticity at the edge of the hole
  - Maximum applied stress = 22.4 ksi
    - Local elastic stress = 70.5 ksi
    - Limited plasticity at the edge of the hole

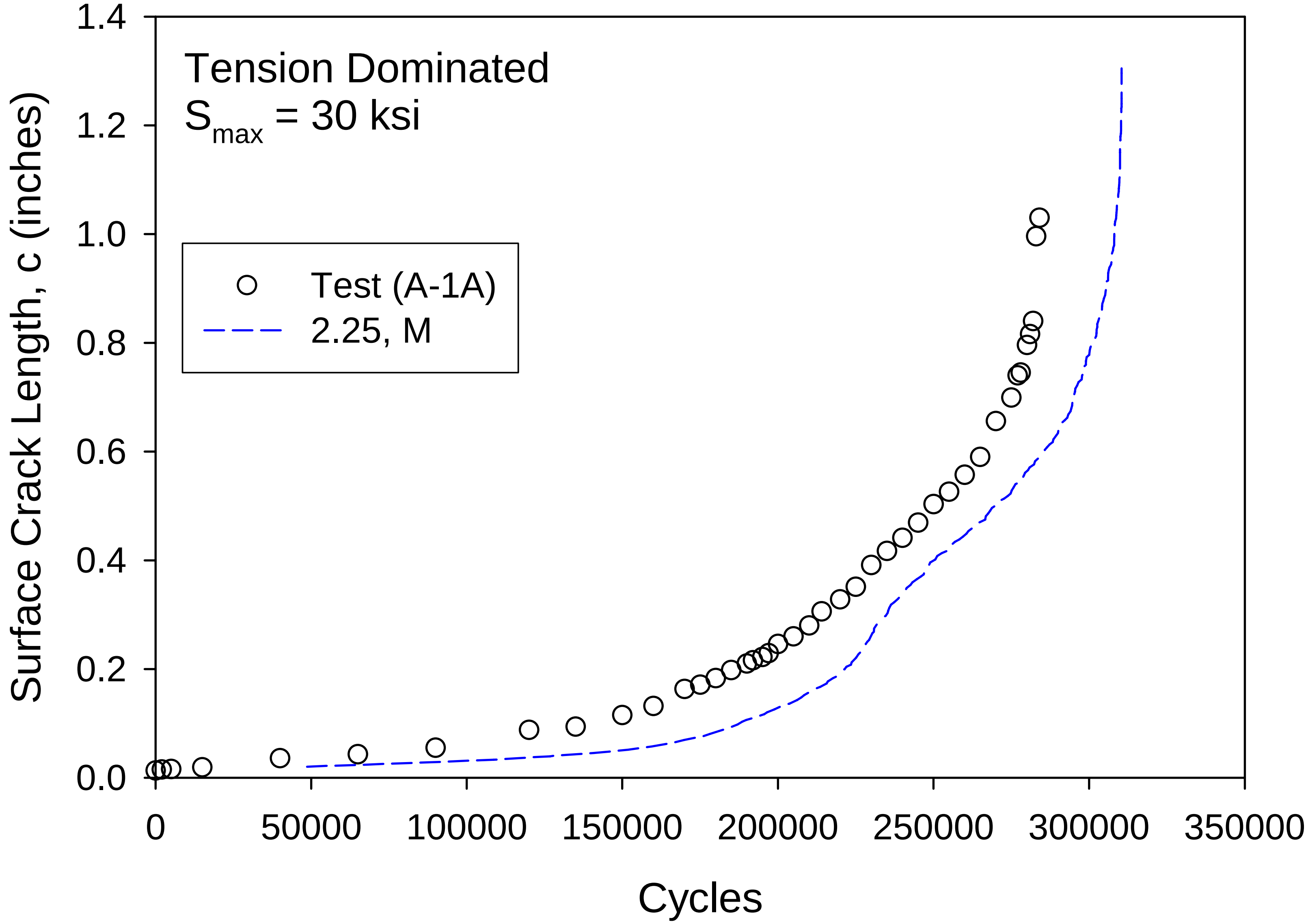
Experimental Evaluation: Load Spectra



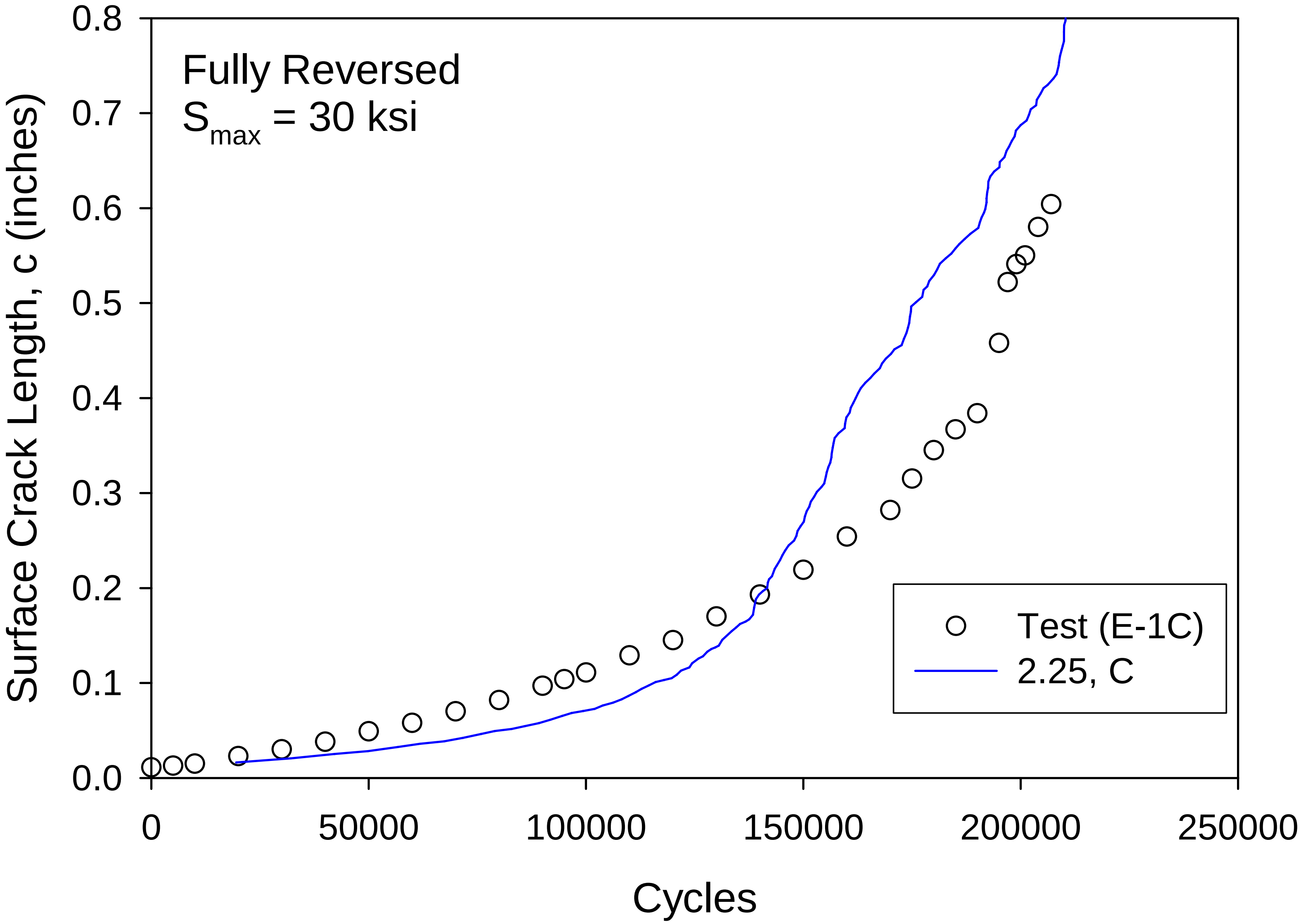




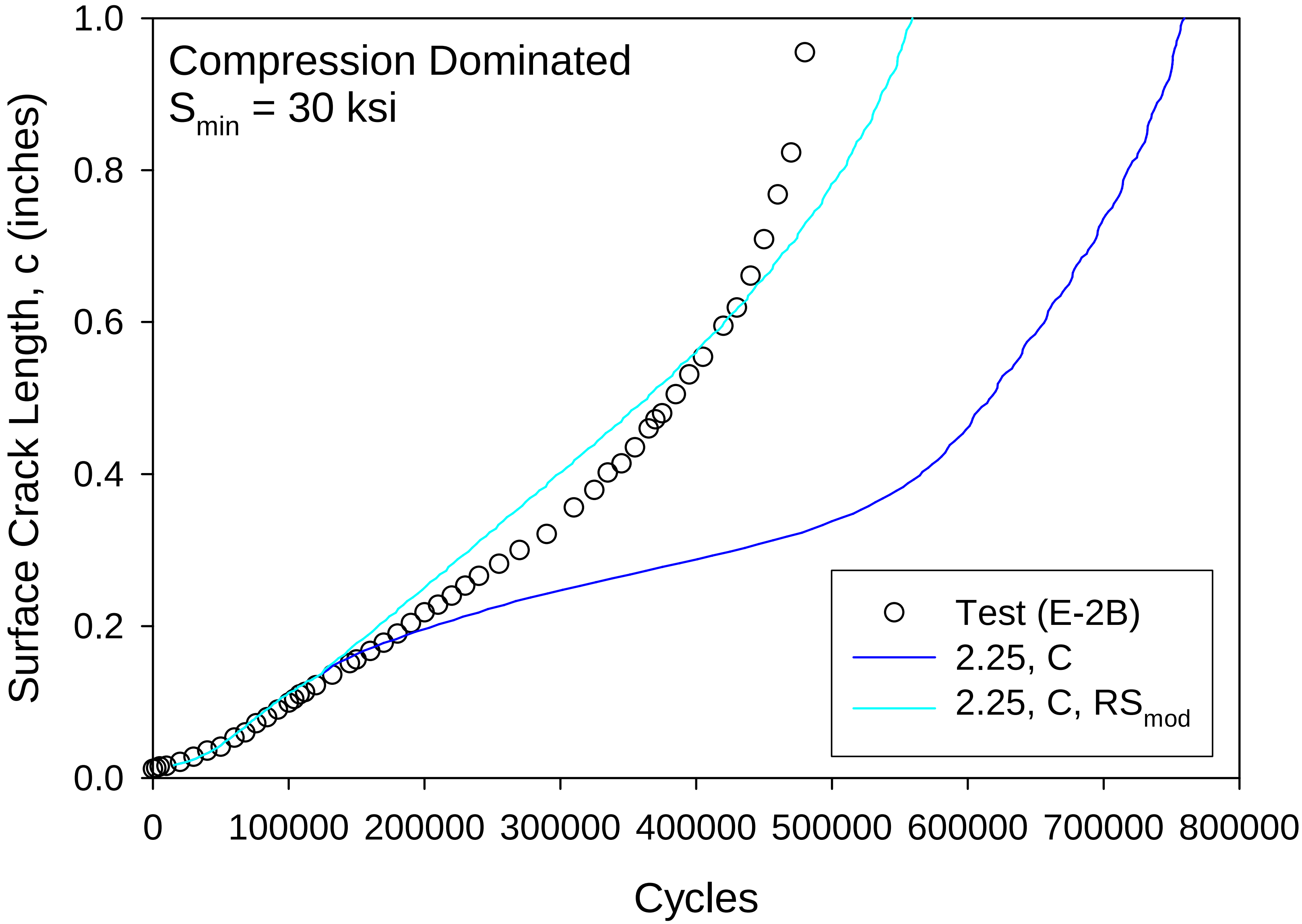
Test Results vs. Analysis: Tension-Dominated, High Plasticity



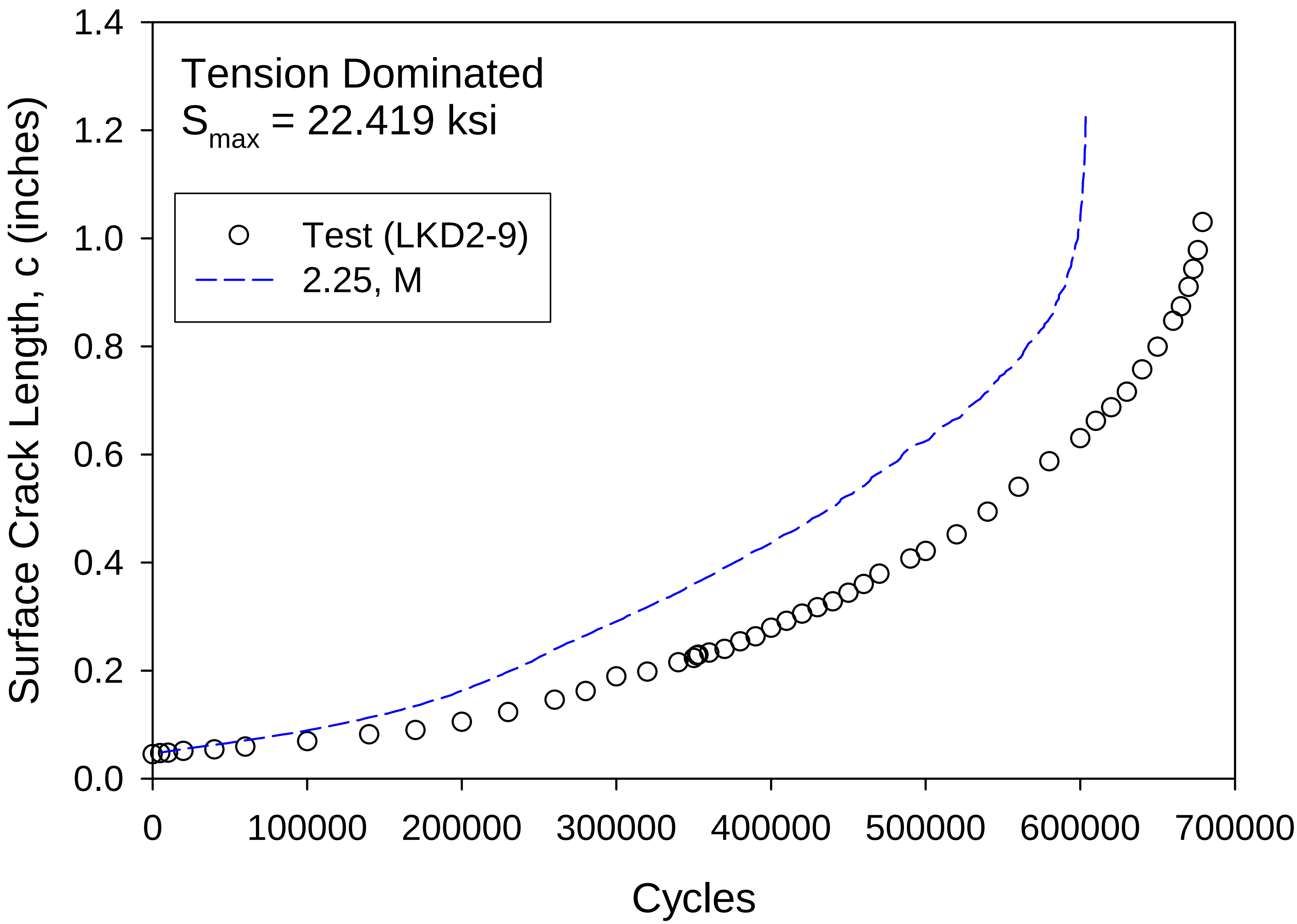
Test Results vs. Analysis: Fully-Reversed, High Plasticity



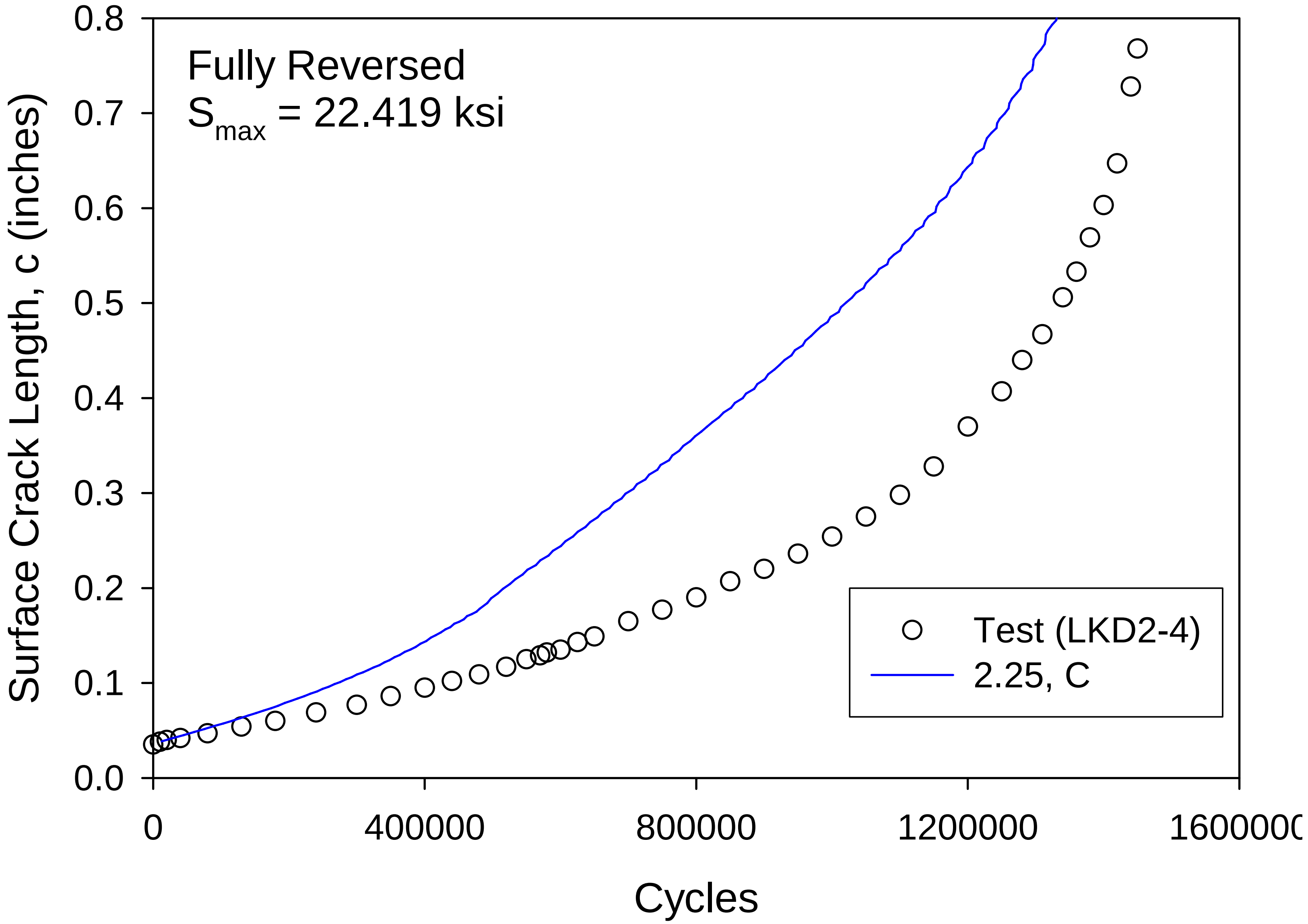
Test Results vs. Analysis: Compression-Dominated, High Plasticity



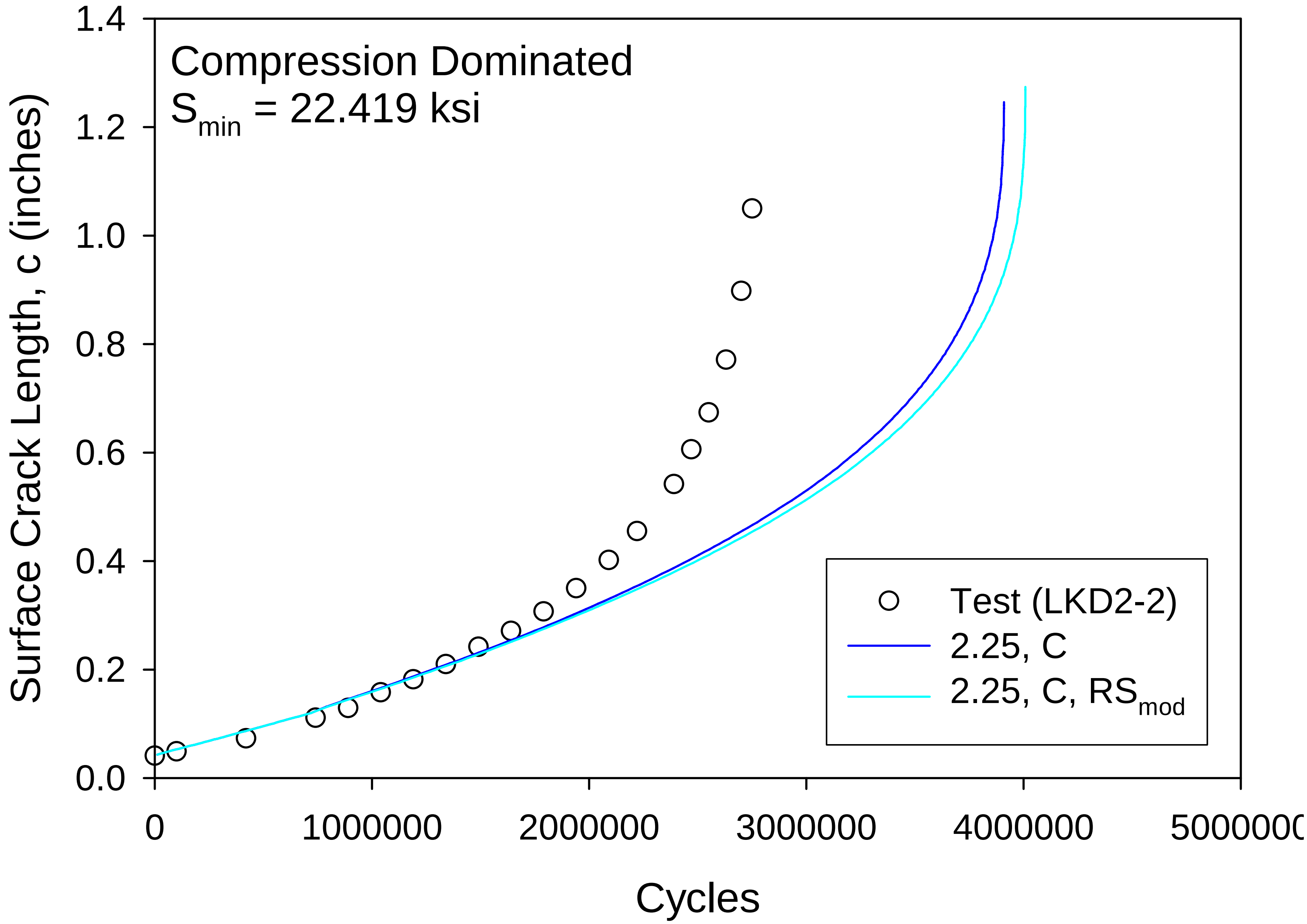
Test Results vs. Analysis: Tension-Dominated, Low Plasticity



Test Results vs. Analysis: Fully-Reversed, Low Plasticity



Test Results vs. Analysis: Compression-Dominated, Low Plasticity



Conclusions

- A new elastic-plastic fatigue crack growth methodology has been developed to address nonlinear effects for cracks growing from stress concentrations
- New NASGRO strip-yield models that include
  - Shakedown analysis
  - New weight function *K* solutions
  - New crack closure formulation
  - J-integral effects

  were generally accurate under
  - Three contrasting spectrum loading conditions
  - Two contrasting load levels