Development of a Probabilistic Fatigue Life Model using AFGROW

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Motivation

- Increasing emphasis on reliable design of aircraft components
  - Design for six sigma
  - Understanding of performance at a specific risk level (e.g. p = 0.01)
- AFGROW has become a commonly used life prediction software (Harter, IJOF, 1999)
  - Applied to structures under spectrum loading (Barter et al., ASIP 2005, Huang et al., TAFM 2005, Zhang et al., IJOF 2003) and fretting fatigue (Giummarra, Trib., 2006)
- Nessus and Unipass are commercially available probabilistic software
  - Can probabilistic software be linked to AFGROW to create a design tool for predicting life of aircraft components?

Objective

- To develop a probabilistic interface for AFGROW using existing probabilistic software
- To demonstrate that the probabilistic interface can accurately and efficiently predict results
  - Modeling and comparison of experimental data

Outline

- Probabilistic interface for AFGROW
- Verification studies
  - CT specimen with variable material properties
  - SENT specimen with variable material properties and initial crack size
- Considerations
  - Various probabilistic methods
  - Sensitivity analysis highlights most important parameters
  - Comparison of Nessus and Unipass

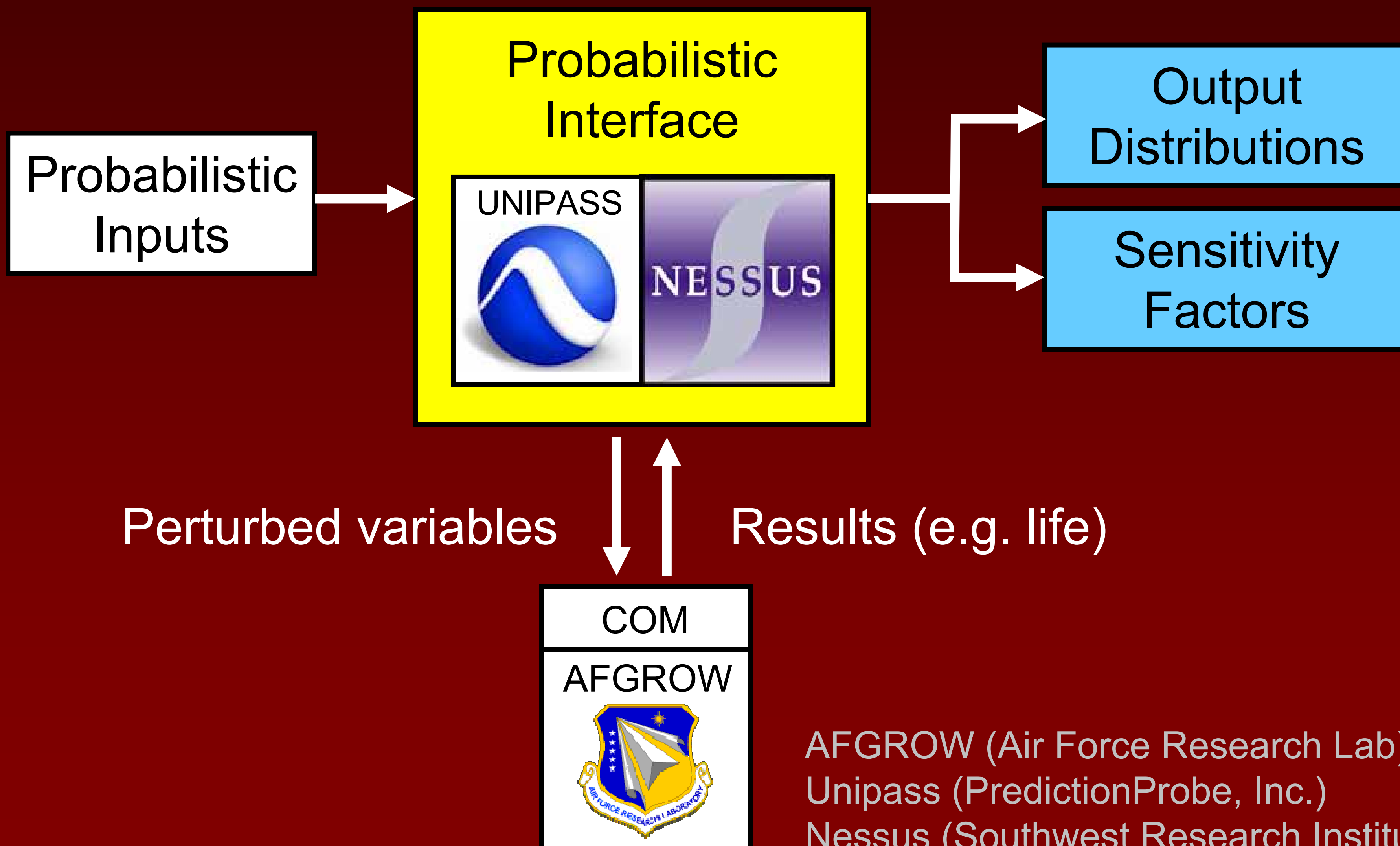
Probabilistic Analysis - Overview

- Many variables affecting fatigue are not constant
  - Material properties have scatter
    - Crack growth rate relation (da/dN versus ΔK)
    - Fracture toughness (K_{IC}) and yield strength (S_Y)
  - Dimensions have tolerances
  - Loading spectrums can vary depending on usage and conditions

General approach

- Represent variables as distributions in order to predict a distribution of performance
  - Variable interaction effects are included

Probabilistic Interface



AFGROW (Air Force Research Lab)
Unipass (PredictionProbe, Inc.)
Nessus (Southwest Research Institute)

AFGROW

AFGROW

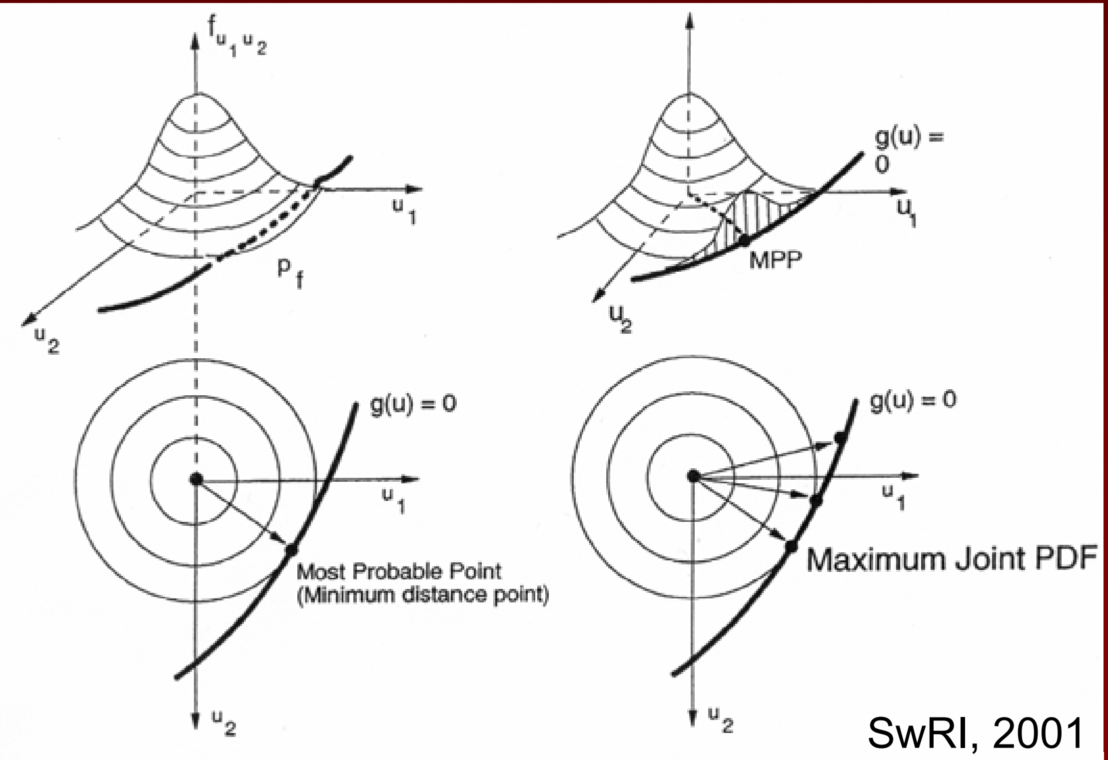
- AFGROW life prediction software
  - Version 4.10.13 used in this study
- Features
  - Efficient weight function based K solutions
  - Crack closure models
  - Repair and inspection
- COM interface allows parametric study of design parameters using Excel
  - Utilized in probabilistic interface

Reliability Methods

- Monte Carlo simulation
  - Randomly generates parameter values from their distributions
  - Evaluates failure criteria, repeat for N trials
- Advantage: simple, robust, guaranteed to converge
- Disadvantage: computationally intensive

Reliability Methods

- Most probable point (MPP) methods
  (Haldar & Mahadevan, Wiley, 2000)
  - Optimization to find MPP
  - Distance to MPP relates to probability



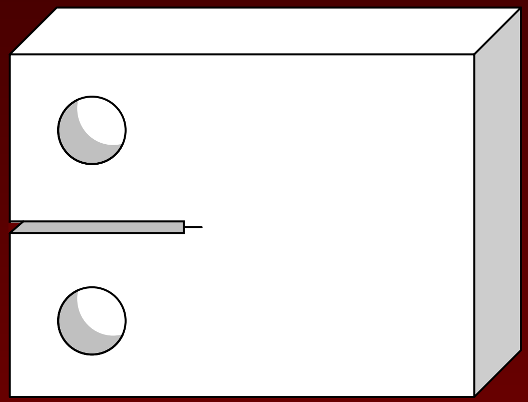
Reliability Methods

- Mean value (MV): approximates MPP by perturbing variables near the mean
  - Number of trials = 1 + (number of variables)
- Advanced mean value (AMV): MV + additional evaluation at the MPP
  - Number of trials = 1 + (number of variables) + (number of p levels)
- FORM: various algorithms based on first order approximation of the performance function
  - Number of trials dependent on convergence
- Advantage: efficient, sensitivity factors
- Disadvantage: approximate, complex to implement, but available in probabilistic software packages

Model Verification #1
CT specimen

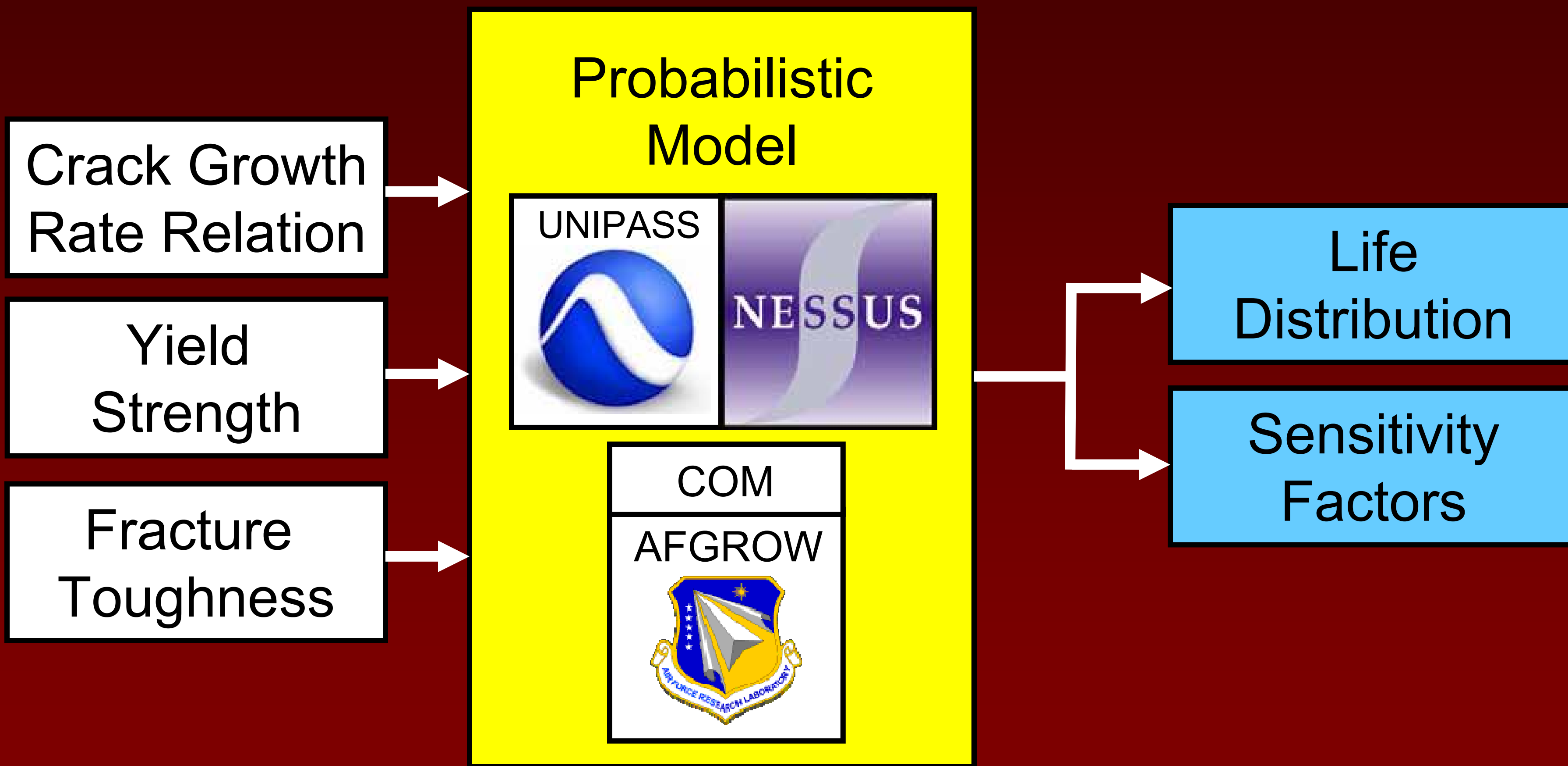
Verification of Model

- Data available for 30 constant amplitude fatigue tests on CT specimens of Al 2024-T351 (Wu & Ni, Prob Eng Mech, 2003)



- Probabilistic model
  - Identical geometry
  - Variability in fatigue crack growth rates, fracture toughness and yield strength
    - Yield strength affects crack closure

Probabilistic Model



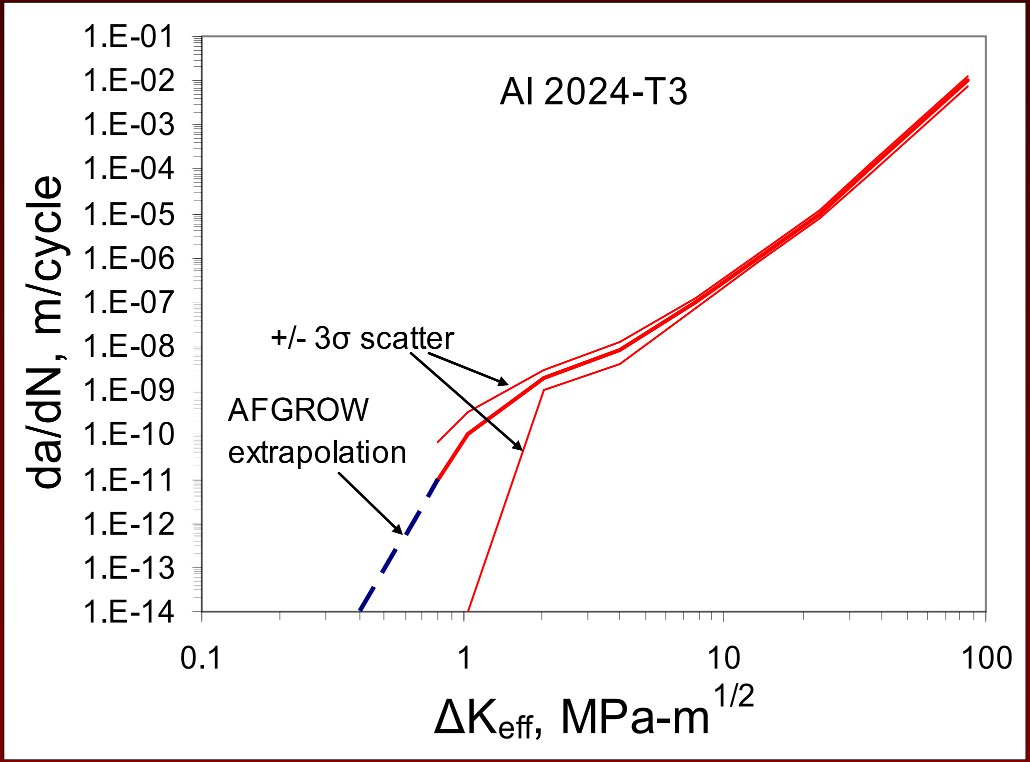
AFGROW Model

- CT geometry based on ASTM Standard E647-93
  - W = 50 mm, B = 12 mm
  - Initial crack size = 15 mm
  - P_{max} = 4.5 kN, R = 0.2
- Modeled with FASTRAN II crack closure model
  - $$\frac{da}{dN} = C_1 \Delta K_{eff}^{C_2} \left[1 - \left(\Delta K_o / \Delta K_{eff} \right)^2 \right]$$
  - C_1, C_2 from da/dN-ΔK_{eff} curve
  - Effective threshold ΔK_o = 0.1 MPa-$m^{1/2}$
- Failure based on exceeding fracture toughness

Crack Growth Rate Relation

- Log-normal distribution for da/dN
- Mean based on piecewise curve (Newman et al., IJOF, 1999)
- Standard deviations from Wang (IJOF, 1999)

- da/dN curve moved from mean curve based on FCGR offset and standard deviation at each ΔK_{eff} value
- Accounts for specimen-to-specimen variation



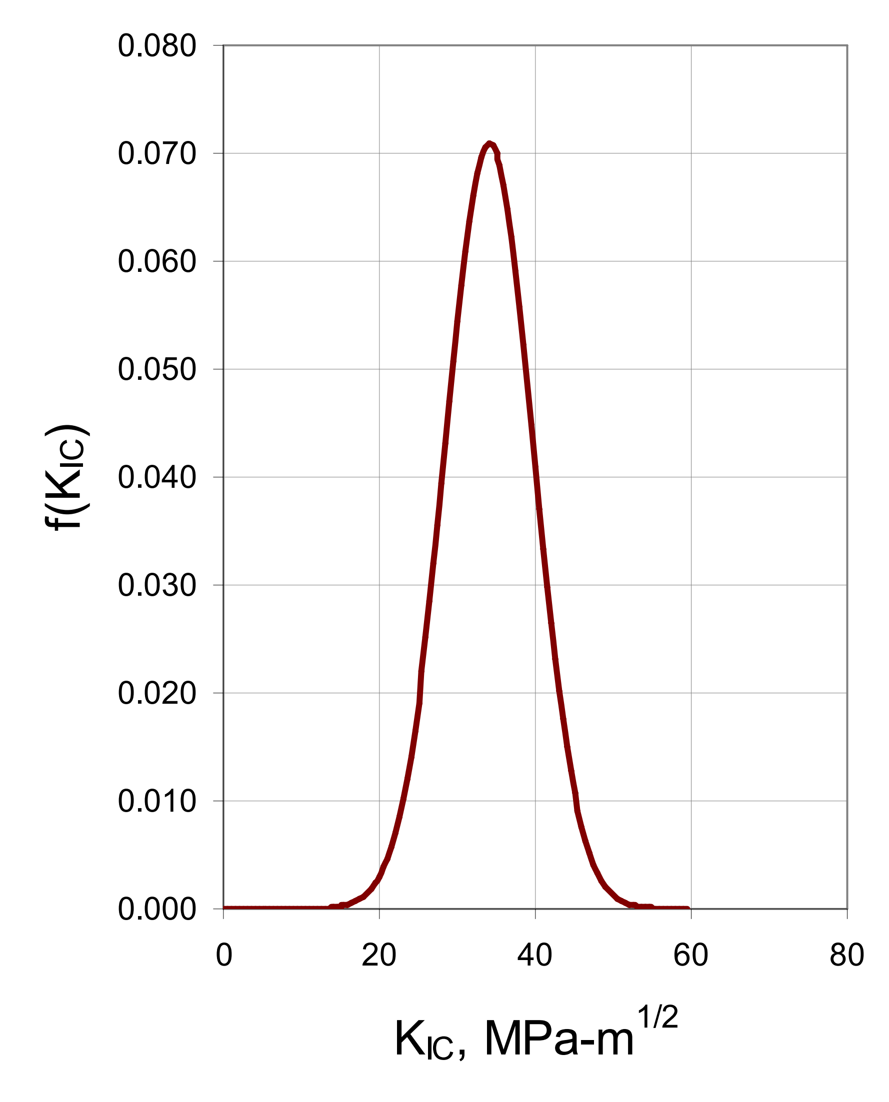
Fracture Toughness

- Al 2024-T351 plate
- K_{Ic}: μ = 34 MPa-m$^{1/2}$

  σ = 5.6 MPa-m$^{1/2}$

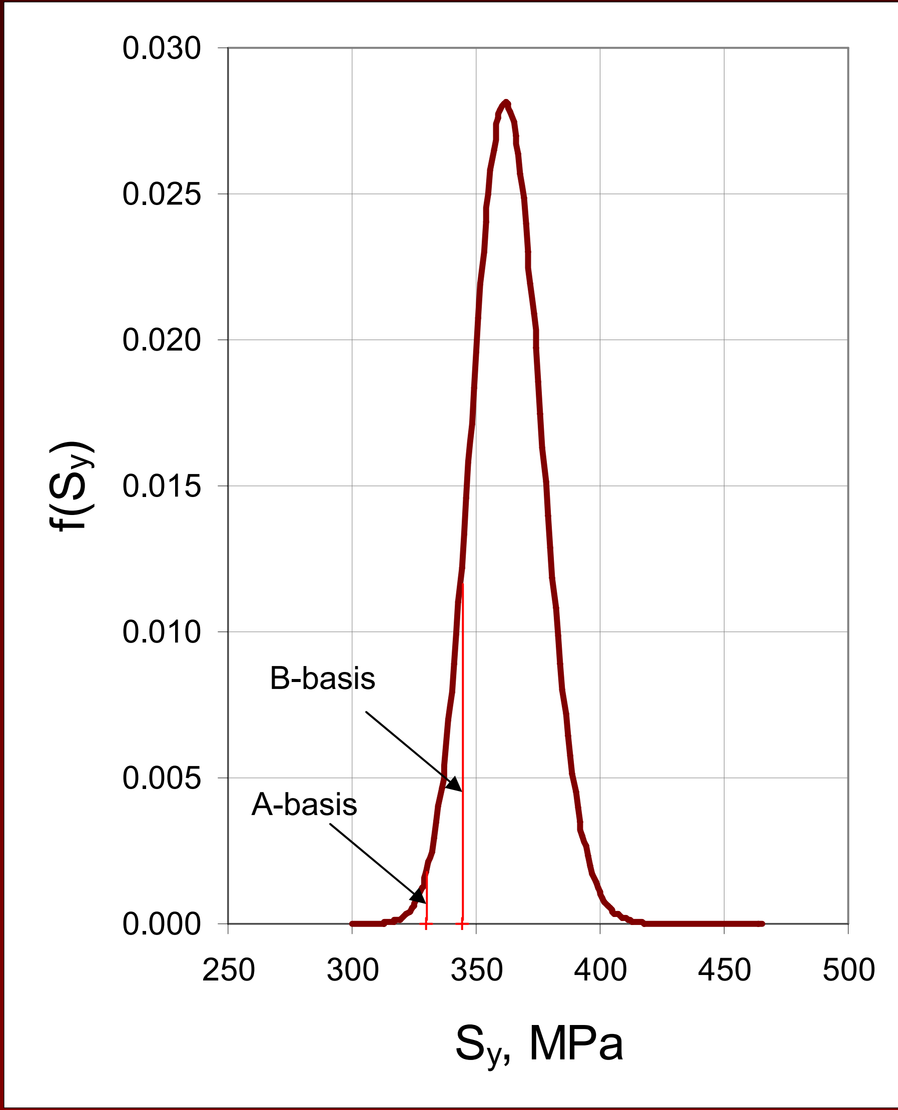
  (MIL-HDBK-5J)

- Assumed normal distribution (White et al., IJOF, 2005; Wang, Eng Fract Mech, 1995)



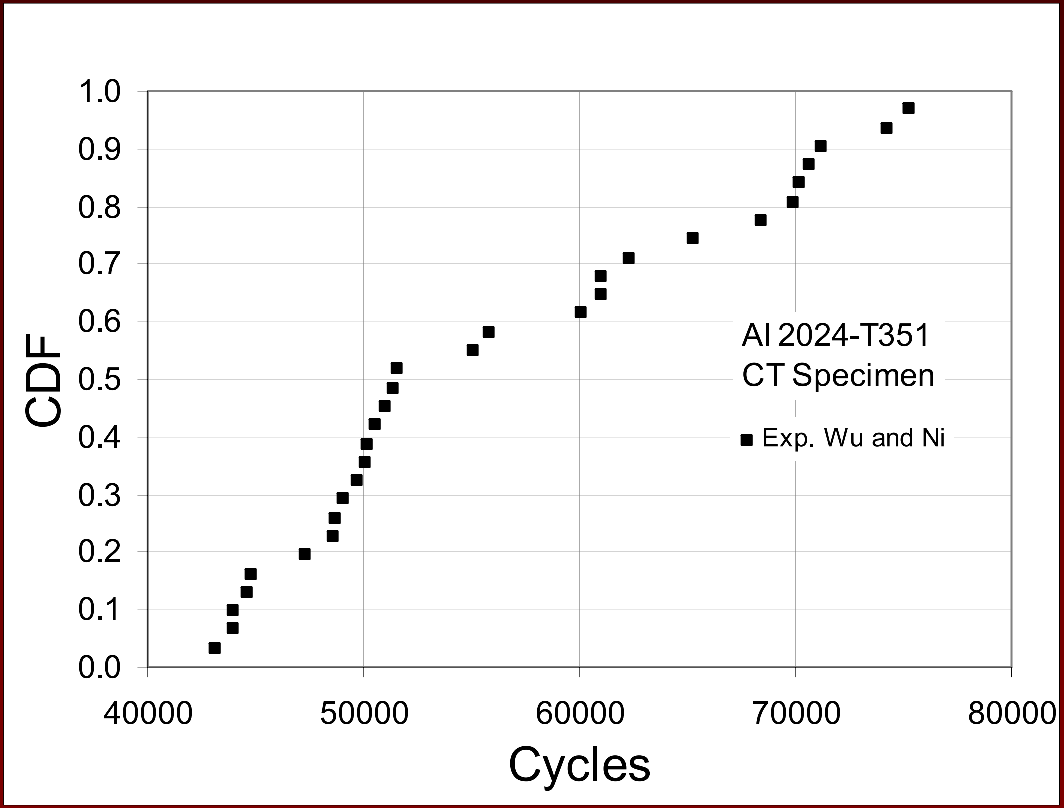
Yield Strength

- Al 2024-T351 plate
- YS: 331 MPa (A-basis)
  345 MPa (B-basis)
  (MIL-HDBK-5J)
- A-basis: 99% of specimens with strength above this value
- B-basis: 90% of specimens with strength above this value
- Assumed log-normal distribution and computed shape (ζ) and scale (λ) parameters



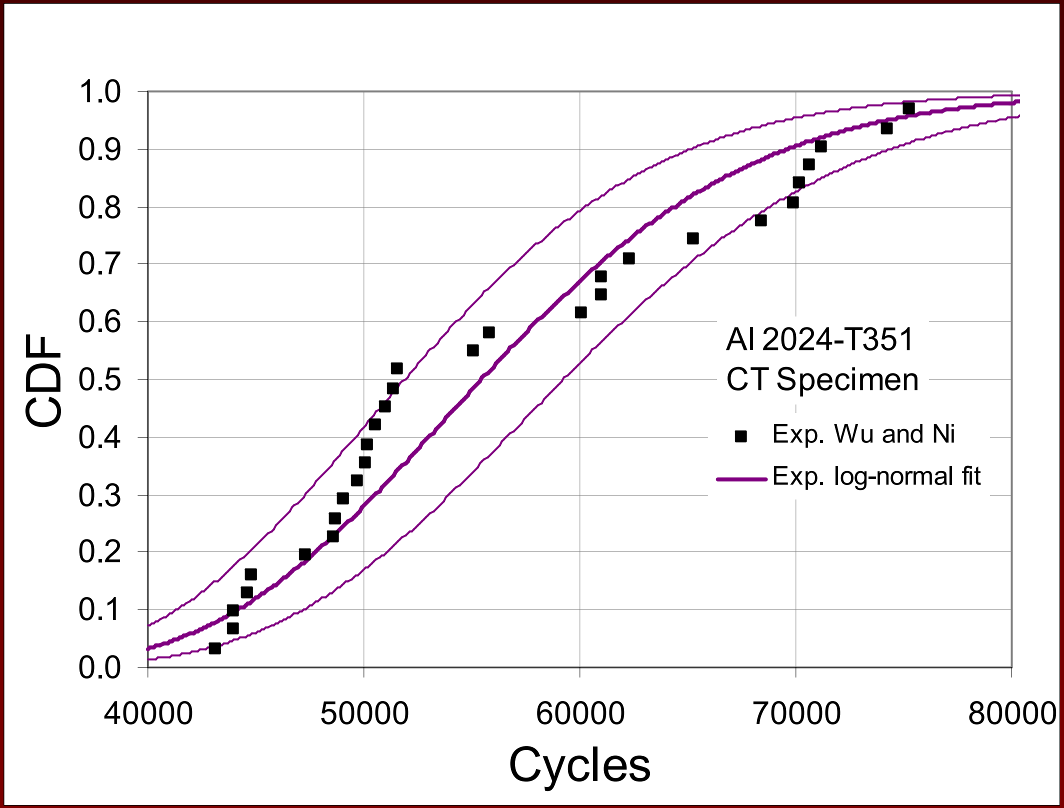
Fatigue Life Distribution

- Experimental data (Wu and Ni, Prob Eng Mech, 2003)
- Life to failure
  - μ = 56,314 cycles
  - σ = 10,231 cycles



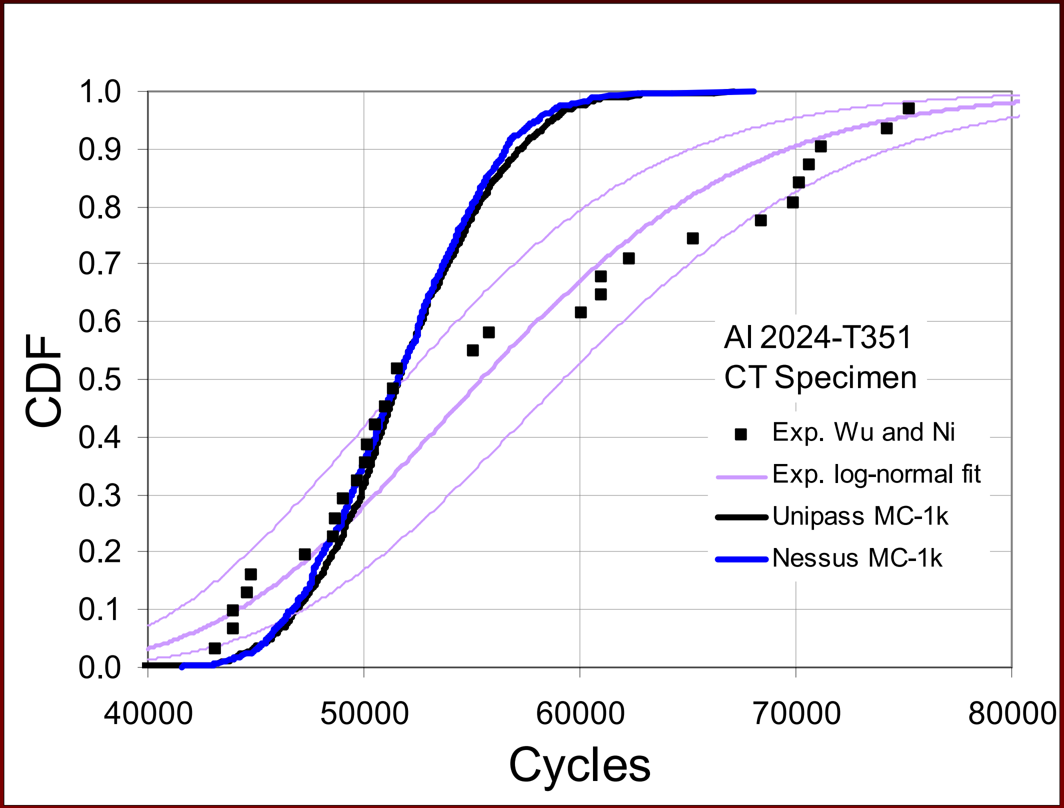
Fatigue Life Distribution

- Fit log-normal distribution to data
  - 5% and 95% bounds on mean curve
- Fit acceptable at 5% significance level (K-S test)



Fatigue Life Distribution

- Monte Carlo simulation
  - 1,000 trials
- Predicted (Unipass and Nessus)
  - μ = 52,000 cycles
  - σ = 4,000 cycles
- Experimental
  - μ = 56,314 cycles
  - σ = 10,231 cycles



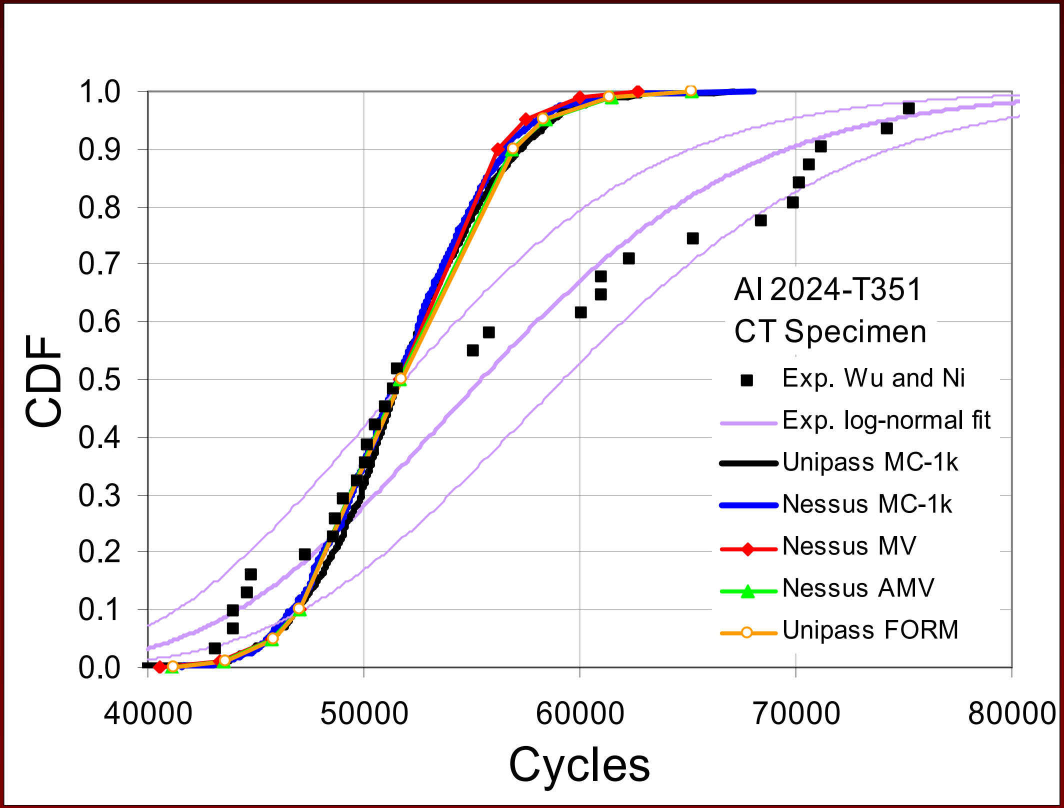
Fatigue Life Distribution

- MV and AMV from Nessus
- FORM from Unipass

- AMV and FORM results within the MC sampling error

- Good agreement for critical shortest lives

- Long-life behavior not accurately modeled
  - Different mechanism?



Predicted Life at $P_f = 0.01$

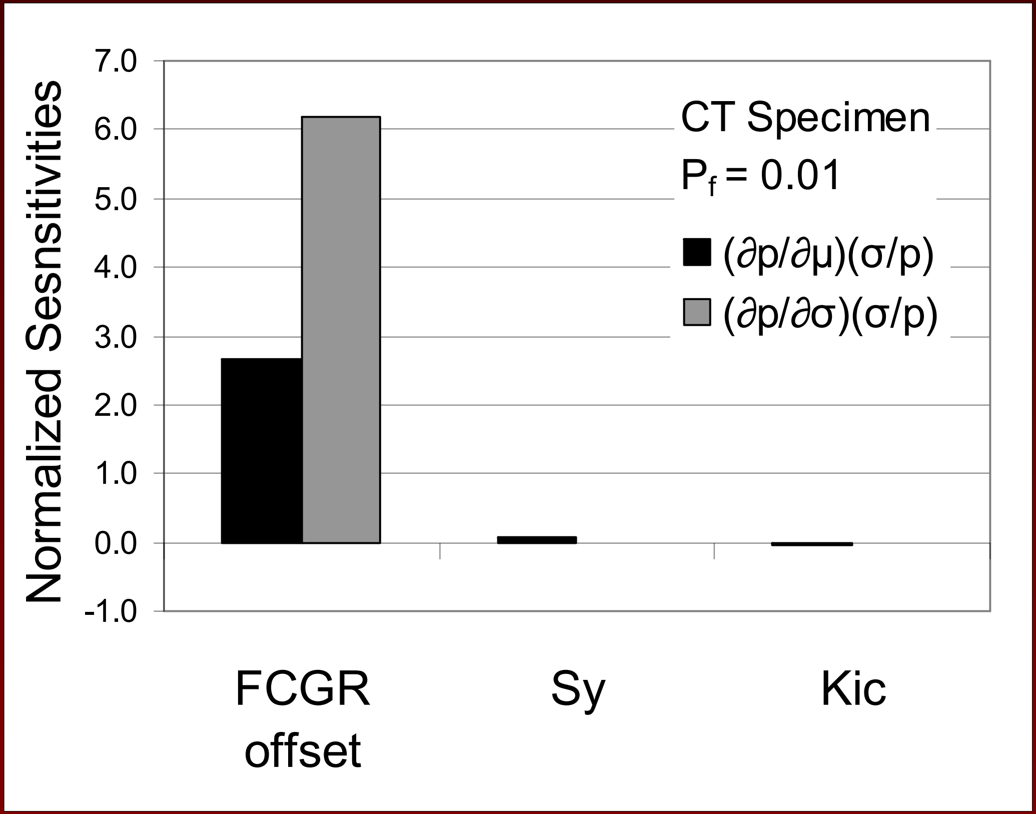
Method	Life (cycles), $P_f = 0.01$	# of Trials	Time
N-MC-1k	42,697	1000	17 hrs
U-MC-1k	43,671	1000	16 hrs
N-MV	43,292	4	4 min
N-AMV	43,495	5	5 min
U-FORM	43,911	8	7 min

N=Nessus, U=Unipass

- AMV, MV and FORM give comparable results to MC in small fraction of time
- Best performance
  - AMV and MV methods in Nessus
  - FORM method in Unipass

Sensitivity Analysis

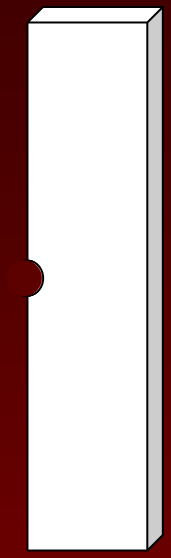
- Sensitivity factors are a measure of relative importance for each variable's contribution to scatter in life with respect to μ and σ
- Variability in CGR relation most important
- S_y and K_{IC} may be modeled as deterministic



Model Verification #2
SENT specimen

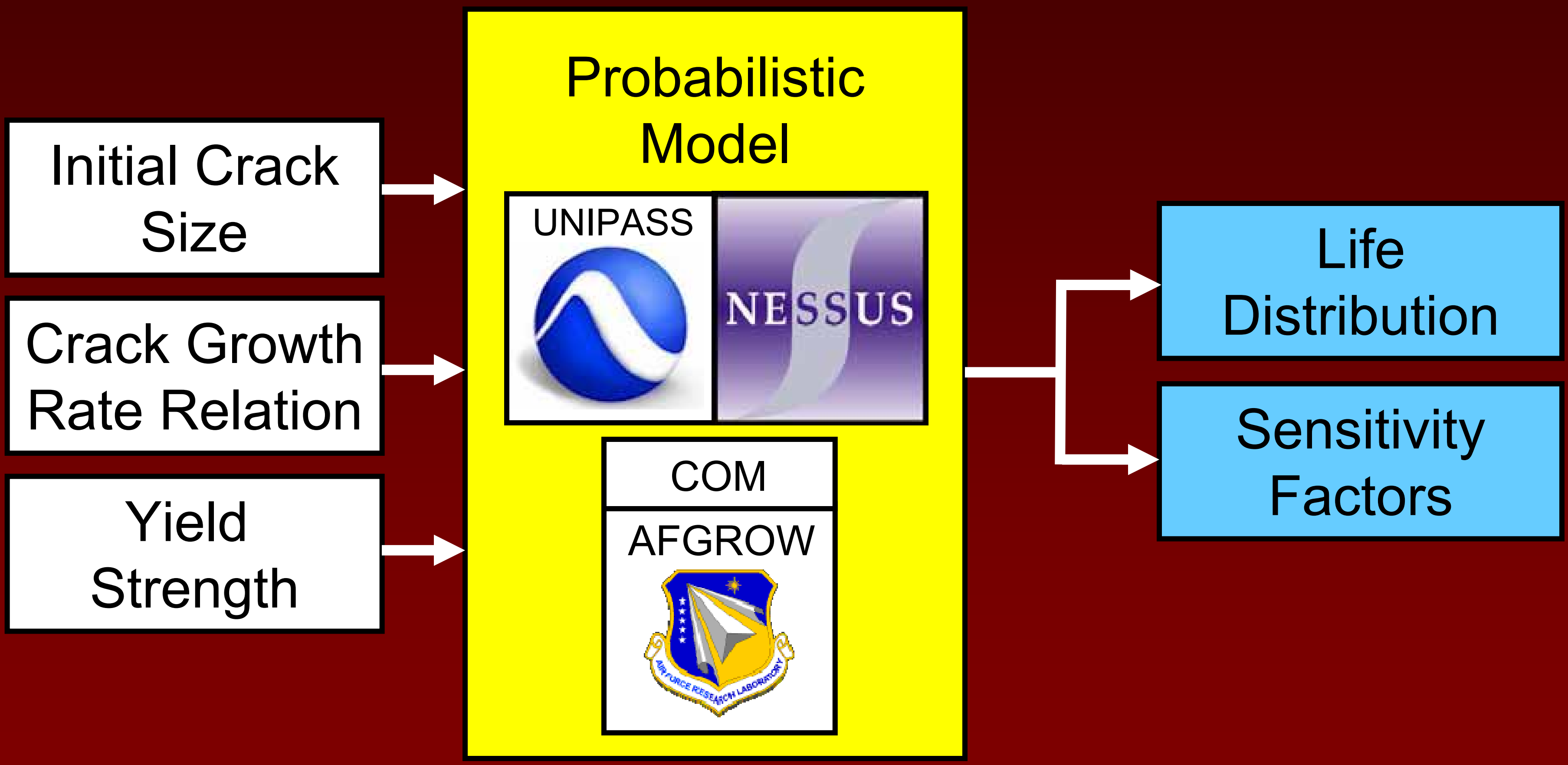
Verification of Model

- Data available for 24 constant amplitude fatigue tests on SENT specimens of Al 2024-T3 (Laz et al., IJOF, 2001; Newman et al., AGARD, 1988)



- Probabilistic model
  - Identical geometry
  - Variability in initial crack size, fatigue crack growth rates and yield strength
    - Initial crack size based on microstructural features
    - Yield strength affects crack closure

Probabilistic Model



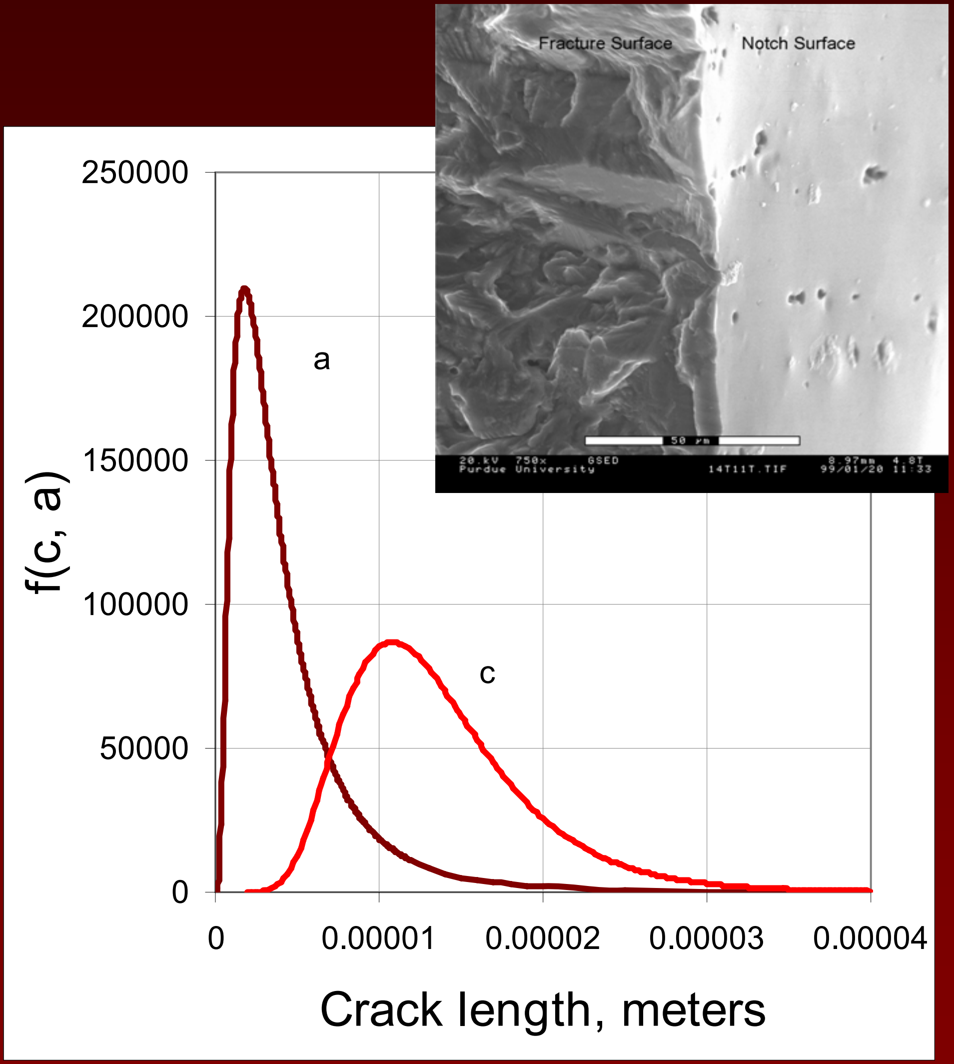
AFGROW Model

- SENT geometry
  - W = 45 mm, L = 203 mm, B = 2.54 mm
  - r = 2.813 mm, K_t = 3.165
  - S_{max} = 120 MPa, R = 0
- Modeled with FASTRAN II crack closure model
  - $$\frac{da}{dN} = C_1 \Delta K_{eff}^{C_2} \left[1 - (\Delta K_o / \Delta K_{eff})^2 \right]$$
  - C_1, C_2 determined with da/dN-ΔK_{eff} curve
  - Effective threshold ΔK_o = 0.1 MPa-$m^{1/2}$
- Failure based on life to breakthrough

Initial Crack Size Distribution

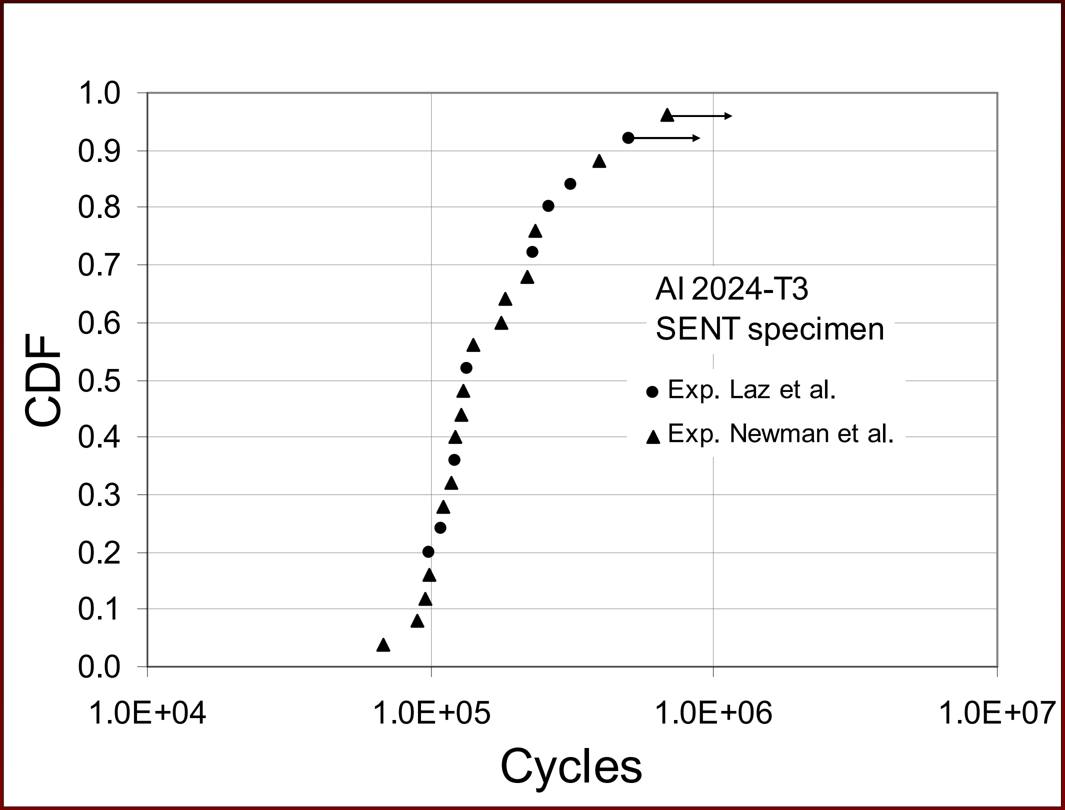
- Based on crack nucleating particles in Al 2024-T3 SENT specimens
  - Measured with replica techniques (Laz et al., IJOF, 2001)

- Log-normal distribution
  - Width 2a: $\mu = 8.95\ \mu m$, $\sigma = 4.10\ \mu m$
  - Depth c: $\mu = 13.6\ \mu m$, $\sigma = 5.58\ \mu m$
  - Correlation coefficient of 0.0359



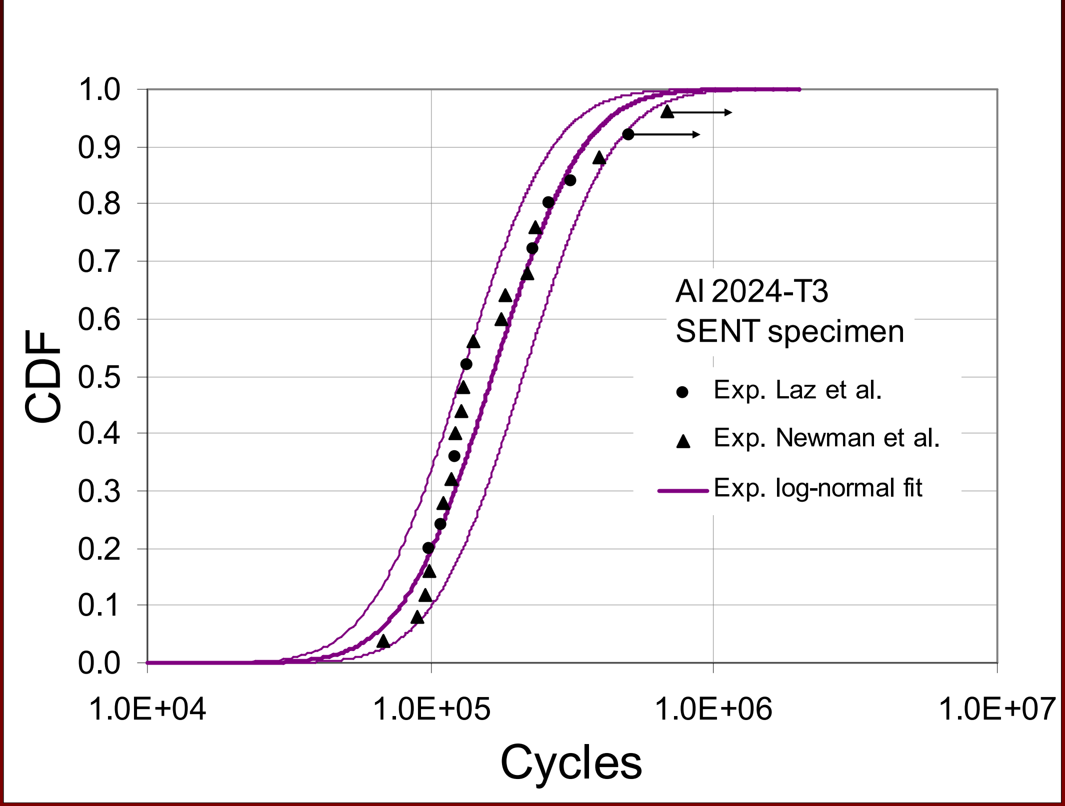
Fatigue Life Distribution

- Experimental data
  - Laz et al. (IJOF, 2001)
  - Newman et al. (AGARD, 1988)
- Life to breakthrough
  - μ = 198,515 cycles
  - σ = 146,200 cycles



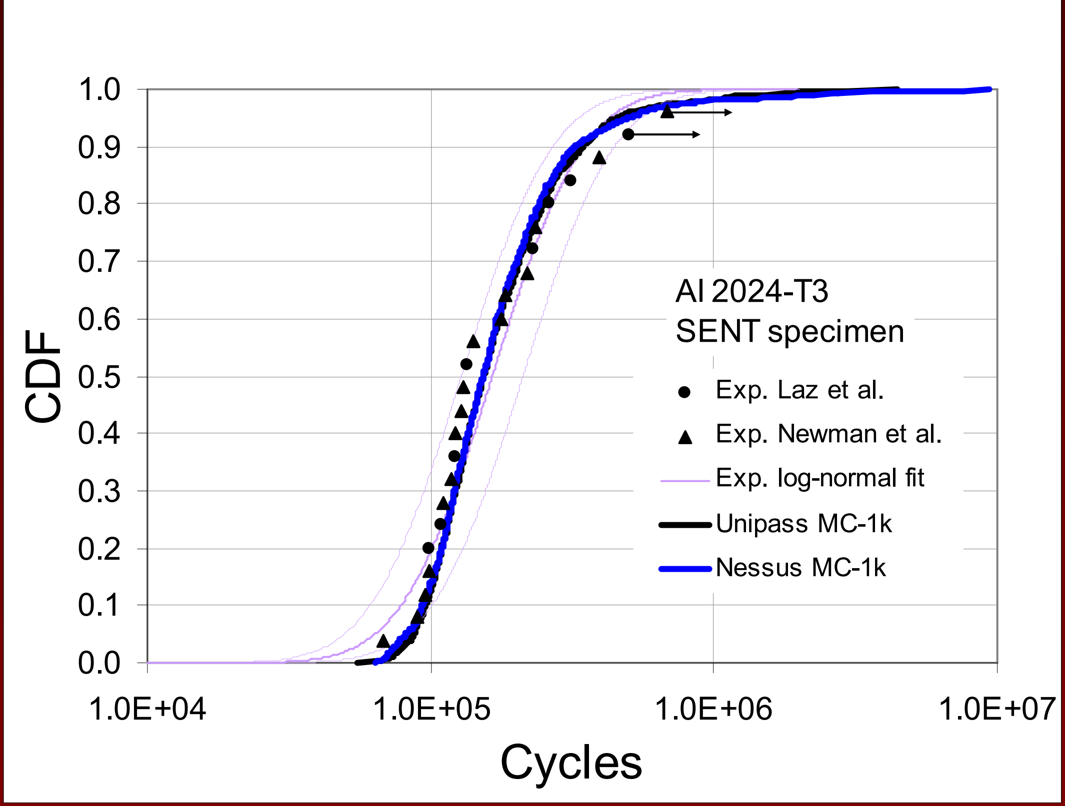
Fatigue Life Distribution

- Fit log-normal distribution to data
  - 5% and 95% bounds on mean curve
- Fit acceptable at 5% significance level (K-S test)



Fatigue Life Distribution

- Monte Carlo simulation
  - 1,000 trials
- Predicted (Unipass)
  - μ = 215,000 cycles
  - σ = 265,000 cycles
- Experimental
  - μ = 198,515 cycles
  - σ = 146,200 cycles

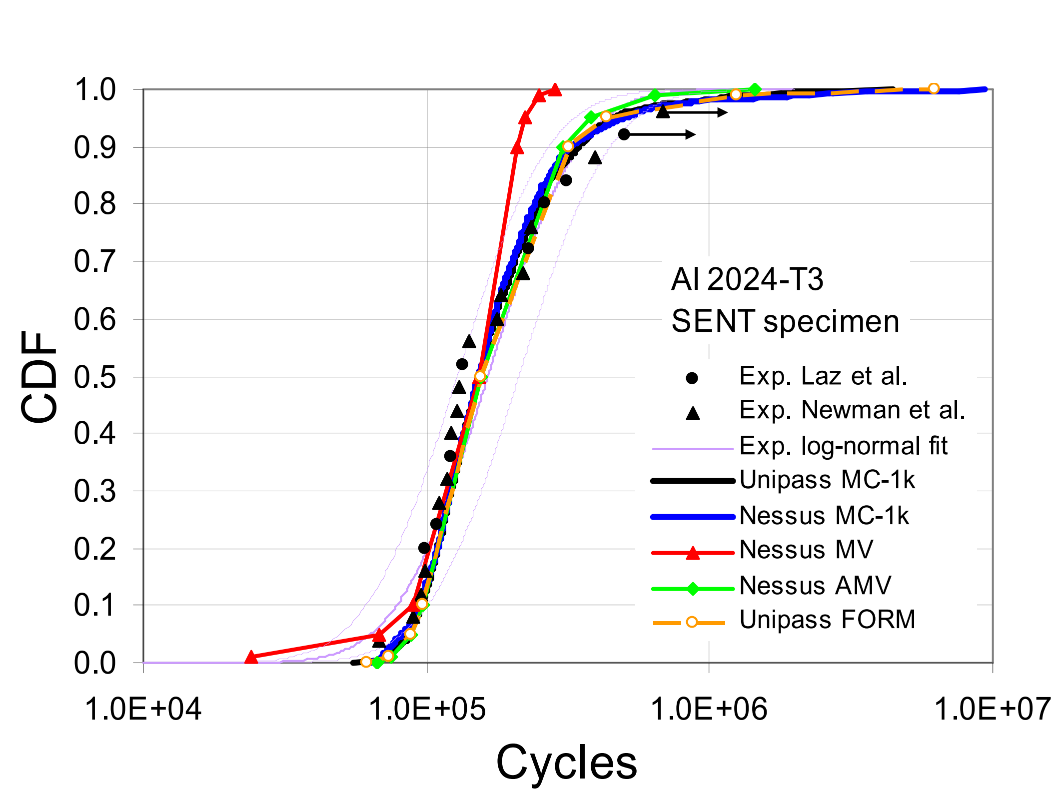


Fatigue Life Distribution

- MV and AMV from Nessus
- FORM from Unipass

- MV less accurate in regions away from mean
  - Limitation of method

- Very good agreement for AMV and FORM with MC



Predicted Life at $P_f = 0.01$

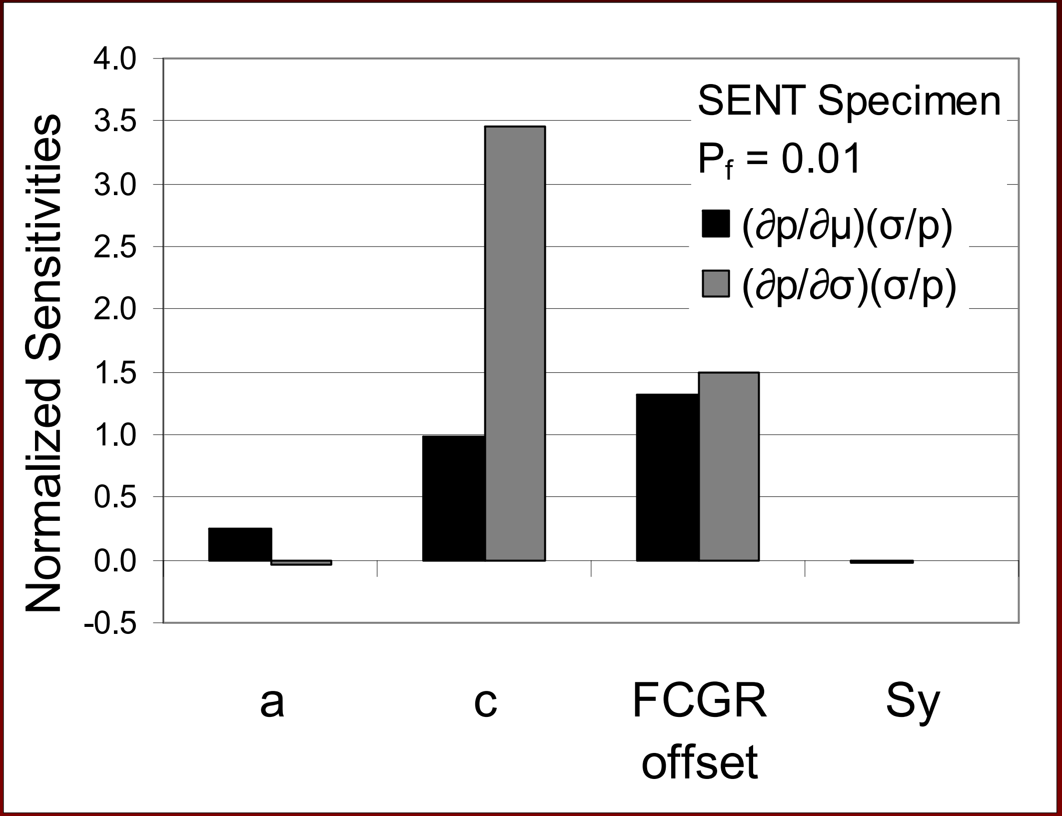
Method	Life to breakthrough (cycles), $P_f = 0.01$	# of Trials	Time
N-MC-1k	68,465	1000	1.3 hrs
U-MC-1k	72,864	1000	1.3 hrs
N-MV	24,035	5	17 sec
N-AMV	75,327	6	20 sec
U-FORM	73,505	10	40 sec

N=Nessus, U=Unipass

- Limitation of MV method - often less accurate in tail regions
- AMV and FORM give comparable results to MC in small fraction of time

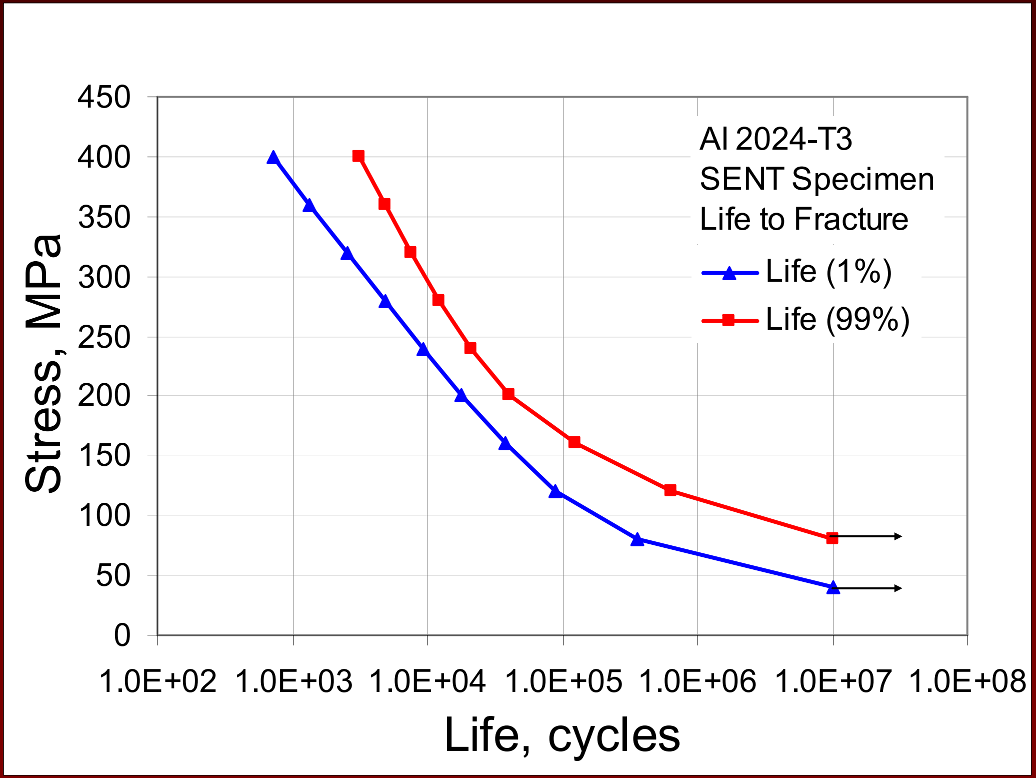
Sensitivity Analysis

- Life most sensitive to amount of variation in initial crack depth (c)
- CGR relation plays important role
- S_y may be modeled as deterministic



Probabilistic S-N Curve

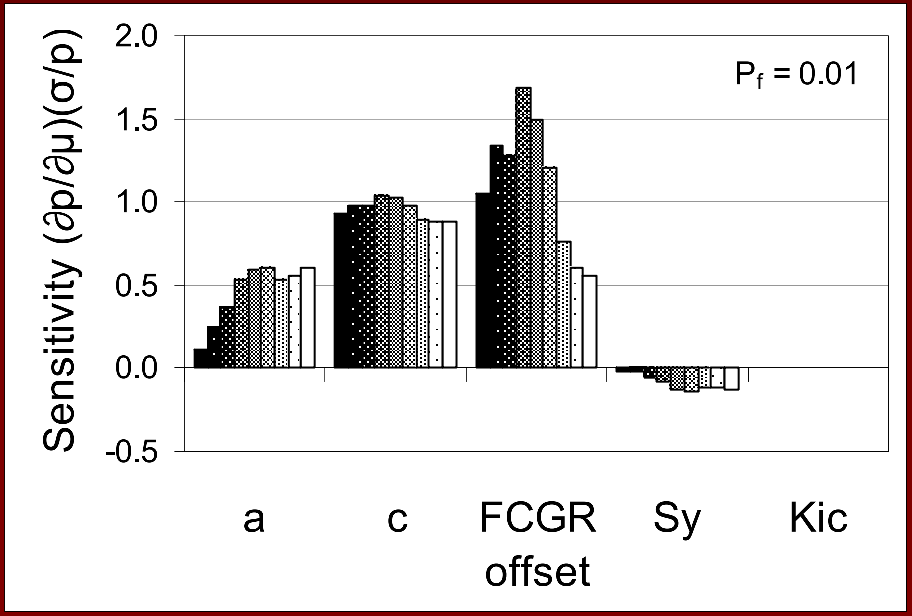
- Fatigue life to fracture for the SENT specimen
- AMV method evaluated at multiple stress levels to compute 1% and 99% bounds
- Useful in design evaluation and risk assessment
- Computation time of 12 minutes

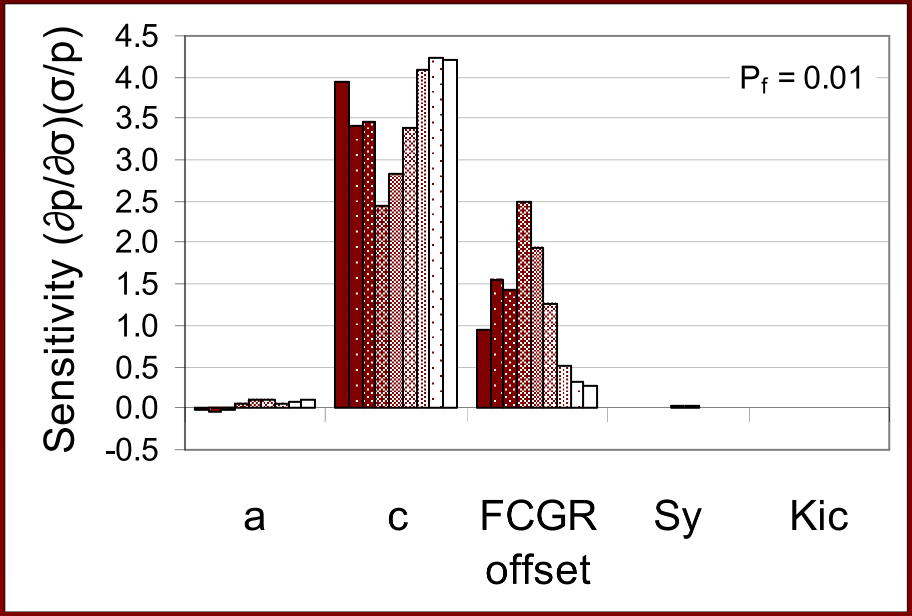


Probabilistic S-N Curve

- Sensitivities depend on stress level (from AMV)
- Life was most sensitive to amount of variation σ in initial crack depth (c)
- CGR relation also an important factor

→ Increasing stress from 80 to 400 MPa





Discussion

- Interface developed to link probabilistic software (Nessus or Unipass) with AFGROW
  - Custom scripting utilized COM interface
  - While demonstrated here for lab fatigue tests, interface can be used with variability in any parameter available in AFGROW
- Efficient probabilistic methods accurately predicted the shortest fatigue lives in both experiments
  - AMV for Nessus, FORM for Unipass
- Probabilistic AFGROW analyses can provide important information for assessing risk of aircraft components
  - Efficient probabilistic methods can provide timely information for decision making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Thank you!
Questions?

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